UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 7 June, 2013 1:30 pm to 3:30 pm

PAPER 72

POLAR OCEANS AND CLIMATE CHANGE

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

In all questions assume that the latent heat of fusion of ice is 336 kJ kg^{-1}, and its specific heat is 2.1 kJ kg^{-1} K^{-1}. The density of sea water (ρ_w) can be taken as 1025 kg m^{-3} and g, the acceleration due to gravity, as 9.81 m s^{-2}.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

(a) Derive the dispersion relation for flexural-gravity waves in a uniform, perfectly elastic, floating ice sheet in deep water. The equation should express the wave number k as a function of ice thickness h, angular frequency ω, ice and water densities, Young's modulus E and Poisson's ratio η. Sketch curves of velocity against wave period which show how phase and group velocities vary in relation to waves in the open sea. Comment on the implications of the group velocity curve having a minimum for wave generation by wind within a continuous ice cover.

(b) On 21 March at 0445 UT a swell signal is observed by a tiltmeter situated at 86°N, comprising a northward-travelling monotonic swell of period 30 s. By 22 March at 2125 UT the swell period has diminished to 14 s and the swell disappears.

(i) Explain why shorter waves are not seen.

(ii) Using open water relationships for wave speeds, describe the rate at which the swell period diminishes with time, and calculate the distance of the source of the swell. What do these results tell you about the likely source of the wave energy?

2

(a) Waves entering the marginal ice zone are attenuated through a scattering process. Describe qualitatively the dependence of the attenuation coefficient on (i) frequency, (ii) floe diameter, (iii) floe thickness. What other decay processes dominate in the case of (iv) very small floes (frazil ice and pancakes much less than a wavelength in diameter) and (v) large continuous ice sheets much more than a wavelength in diameter?

(b) When an off-ice wind acts on the marginal ice zone, the ice edge floes are stripped away from the ice margin in the form of parallel bands. By using the result that the wave radiation force F on a unit wavefront of a reflective floating object is given by

$$F = \rho_w g(a^2 + r^2 - t^2)\,,$$

where a, r and t are amplitudes of incident, reflected and transmitted waves, describe the mechanism by which the radiation pressure of short waves in initially random openings organises the floes into coherent bands, acting against the force due to swell. Show the distribution of wave energy inside the band and in the polynyas on either side of it. Explain the evolution of initial bands into composite bands and explain the factors that limit the number of bands that can form.

If the reflection is only partial, the amplitude reflection coefficient at each floe R leads to a compressive stress within the band which can be expressed as $(-dF/dx)$ where F is wave radiation force per unit wavefront. Show that this stress is given by

$$(-dF/dx) = \rho_w g p R^4 a^2 / 2d\,,$$

where d is floe diameter, a is wave amplitude and p is fractional ice cover.

For a band made of floes of diameter 20 m in waves of 0.1 m amplitude, calculate the wave radiation force per unit wavefront assuming perfect reflection, and the compressive stress within the band assuming $R = 0.5$ and $p = 1$.

What do these figures suggest about the stability of the band?

3

(a) Describe how pressure ridges and shear ridges are formed in sea ice.

(b) It is found experimentally that the distribution of pressure ridge drafts h in sea ice is a negative exponential when h exceeds a low cutoff h_0. This distribution can be expressed as a number density $n(h)$ of ridges per unit distance along a track as

$$n(h)dh = B\exp(-bh)dh\,,$$

where B, b are parameters which can be expressed in terms of the mean draft h_m, the cutoff h_0, and the total number of ridges per unit distance μ. Likewise it is found that the probability density function $g(h)$ of ice draft is also a negative exponential with a similar decay parameter, and can be expressed as

$$g(h)dh = A\exp(-bh)dh\,.$$

Relate B and b to h_m, h_0 and μ. On the assumption that ridged ice is composed of pressure ridges of congruent triangular shape, with mean along-track slope angle δ, relate parameter A to B, b and the slope angle δ. In practice, what range of slope angles are found? Do the slopes and underwater shapes of first- and multi-year ridges differ?

(c) If ice having these ridging characteristics is driven at average velocity V by wind and current into shallow water of depth D, give an equation for the rate of scouring of the seabed at a given point due to keels passing with a draft greater than D, in terms of D, V, h_m, h_0 and μ.

(d) If ridges governed by the above distributions were positioned at random and at random orientations on an ice surface, what is the expected distribution of spacings between successive ridges along a line? The actual observed distribution is a lognormal. Write down the equation of the probability density function of a lognormal which has a threshold parameter θ (i.e. X has a lognormal distribution if the random variable $Z = \ln(X - \theta)$ is normally distributed). What is the physical meaning of the threshold parameter in the context of sonar profiles of sea ice ridges? Given the occurrence of lognormally distributed variables in nature, does this tell us anything about the ridging process?

4

(a) A rapid retreat of summer Arctic sea ice has been observed in recent years, a process described as the "Arctic death spiral". (i) Describe the rates at which the area, thickness and composition of summer ice have changed since 1980. (ii) Give reasons why this change may be accelerating and why under current conditions the process is irreversible.

(b) An aspect of the retreat is warming of the seabed on Arctic shelves. In late summer a sea surface temperature of 7°C was detected by satellite in the southern Chukchi Sea, latitude 71°N.

  (i) Assuming that the water depth is 50 m and the water column is isothermal, calculate the heat content of the water column per square metre of sea surface, relative to the freezing point of −1.8°C. If this heat were to advect under ice-covered areas, how much ice could be melted?

  (ii) Assess the total amount of solar heat that could have been absorbed by the newly ice-free ocean during the summer months. Assume the sea surface is ice free from June to August, that the albedo of open water is 0.1 and make reasonable assumptions about sun angle variability (solar constant is 1366 W m^{-2}). Does this imply that it is reasonable to assume that the water column could have been at 7°C as far as the seabed? If not, what other explanation is there for the observed surface temperature?

(c) The release of methane gas plumes from shelf seas is believed to be triggered by seabed warming. What other sources of Arctic methane have been identified? Why is methane regarded as a powerful greenhouse gas? Describe one other direct feedback of sea ice retreat upon global climate.

END OF PAPER