UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 7 June, 2013 9:00 am to 12:00 pm

PAPER 55

ORIGIN AND EVOLUTION OF GALAXIES

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Use the shape and evolution of the observed power spectrum of the spatial distribution of matter in the late-time Universe (redshift $z < 100$) to explain what is meant with hierarchical galaxy formation. Explain when and how this shape was established, and how it relates to the shape of the power spectrum of spatial fluctuations of the energy density in the Early Universe.

Explain the difference between warm and cold dark matter, and how this affects galaxy formation.

Explain what happens, when density fluctuations in the Universe turn "non-linear", and what is meant by the virial temperature of a dark matter halo.

Explain what happens to the baryons in a dark matter halo undergoing non-linear collapse, if the gas

i) cannot cool below the virial temperature of the dark matter halo,

ii) can cool below the virial temperature of the dark matter halo.

Describe the most important cooling processes for gas at temperatures,

i) $T < 10^4$K,

ii) $10^4 < T < 10^6$K,

iii) $T > 10^6$K.

Explain the current understanding of why the efficiency of turning baryons into stars appears to peak in dark matter halos with a mass of $\sim 10^{12} M_\odot$ and to decrease rapidly for halos with larger and smaller mass.

2

Assume the Universe to be Einstein-de-Sitter and matter dominated with Hubble constant $H(t)$ and background density $\bar{\rho} = 3H^2(t)/8\pi G$. Consider a spherically symmetric density perturbation with radius R and enclosed mass M. Show that the parametric solution

$$R = A\,(1 - \cos\theta), \qquad t = B\,(\theta - \sin\theta), \qquad (*)$$

solves the equation of motion of the mass shell for suitable values of the constants A and B. Sketch this solution for a suitable range of θ and express A and B in terms of the radius and time at maximum expansion, $R_{\rm max} = R(t = t_{\rm max})$.

By comparing to the evolution of a spherical homogeneous region of the Universe with the same enclosed mass and density equal to the mean background density $\bar{\rho}$ show that at the time of maximum expansion the overdense region is denser by a factor $\rho/\bar{\rho} = (3\pi/4)^2$.

Assume that the object formed during the collapse reaches virial equilibrium by the time the solution (*) reaches $R(t = t_{\rm coll}) = 0$. Calculate the ratio $\rho_{\rm virial}/\bar{\rho}(t_{\rm coll})$. Explain how this can be used together with an estimate of mass and virial radius of a collapsed object to estimate it's redshift of formation.

3

i) A supermassive black hole discovered at redshift $z = 8$ has been measured to have a mass of $3 \times 10^8 M_\odot$. Assume that the black hole has grown by continuous spherical accretion of gas at the Eddington accretion rate from an initial mass M_0.

Starting from the balance between gravitational and radiative force on the accreting gas show that the black hole mass will have grown as

$$M_{\rm bh} = M_0 \exp[t/t_{\rm e-fold}], \qquad \text{with} \qquad t_{\rm e-fold} \approx \epsilon_{\rm r} \frac{c\, \sigma_{\rm T}}{4\pi G\, m_p} \approx 4.5\, \epsilon_{\rm r}\, 10^8 {\rm yr},$$

where $\epsilon_{\rm r}$ is the radiative efficiency of accretion, $\sigma_{\rm T}$ is the cross section for Thomson scattering and $m_{\rm p}$ is the proton mass.

Compare the time for growth from $M_0 = 30 M_\odot$ to the observed mass at $z = 8$ with the age of the Universe at that time. Assume $\epsilon_{\rm r} = 0.1$, and $\Omega_{\rm mat} = 0.25$, $\Omega_{\rm bar} = 0.05$ and $h = 0.7$ for the present-day values of matter density, baryon density and Hubble constant and comment on whether the black hole could have grown to its observed mass in that way.

ii) Consider the innermost $3 \times 10^9 M_\odot$ of baryons at the centre of a spherical dark matter halo which has collapsed at $z = 8$ and has total mass $3 \times 10^{12} M_\odot$ within the virial radius $r_{\rm vir} = 53$kpc.

Estimate the radius $r_{\rm d}$ and rotational velocity v_d at which these baryons settle into angular momentum support at the centre of the dark matter halo if the specific angular momentum is conserved. Assume that the rotational velocity at the virial radius is 10 percent of the virial velocity and that the distribution of the specific angular momentum of baryons and dark matter are the same and can be described by the mass with specific angular momentum less than j, as $M(< j) = M_{\rm tot}\, (j/j_{\rm vir})$, where $j_{\rm vir}$ is the specific angular momentum at the virial radius. You can also assume that there is no dark matter at $r < r_{\rm d}$. State and explain any additional assumptions you have made.

[You may find the following expression helpful: $3 \times 10^9\, G\, M_\odot \approx 1.3 \times 10^4\, ({\rm km/s})^2\, {\rm kpc}$]

4

i) With the Press–Schechter ansatz, the mass fraction of the matter density in the Universe in collapsed objects with mass greater than M at redshift z is,

$$f(>M, z) = \mathrm{erfc}\left(\frac{\delta_c(z)}{\sqrt{2}\sigma(M, z=0)}\right) \quad \text{with} \quad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp\left(-t^2\right) \mathrm{d}t.$$

Explain the meaning of $\delta_c(z)$ and $\sigma(M, z=0)$.

Assume the power spectrum describing the Gaussian density fluctuations can be approximated as a power law $P(k) \propto k^n$. Using the Press–Schechter ansatz a differential mass function of collapsed objects of the form

$$n(M, z)\,\mathrm{d}M = A\,M^{\alpha} \exp\left[-(M/M_*(z))^{\beta}\right] \mathrm{d}M$$

can be derived, where $M_*(z)$ is a characteristic mass that evolves with time/redshift. Calculate A, α, β and $\sigma(M_*)$.

ii) Assume that star formation in collapsed objects with mass larger than M_{min} produces 3000 ionizing photons per "collapsed baryon" and that the hydrogen in the Universe is fully re-ionized once 3 ionizing photons for each baryon in the Universe have been produced. The amplitude of density fluctuations with enclosed mass $64M_{\mathrm{min}}$ at redshift $z = 3$ has been measured to be $\sigma(M = 64M_{\mathrm{min}}, z = 3) = 0.5$.

At which redshift has the hydrogen in the Universe become fully re-ionized? Assume that the power spectrum slope is $n = -2$ on the relevant mass scale.

Assume that the objects with mass M_{min} collapsing at $z = z_{\mathrm{reion}}$ reach virial equilibrium without disspating any energy and have a virial velocity of 9 km/s. Estimate the typical comoving distance $\bar{l} = [n(M)M]^{-1/3}$ between such objects. Assume $\Omega_{\mathrm{mat}} = 0.25$ and $h = 0.72$ for the present day values of matter density and Hubble constant.

[You may find the following approximate values helpful: $(4.64^2/\pi)^{-1/6} \approx 0.75$, and for the inverse of the complementary error function, $\mathrm{erfc}^{-1}(10^{-3}) \approx 2.33$.]

END OF PAPER