MATHEMATICAL TRIPOS Part III

Monday, 3 June, 2013 1:30 pm to 3:30 pm

PAPER 43

SUPERSYMMETRY

Attempt no more than ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Answer both parts (a) and (b), which are unrelated.

(a) The algebra of the Lorentz group $SO(1,3)$ can be written in terms of generators $M_{\mu\nu}$ as

$$\Big[M_{\mu\nu}\ ,\ M_{\rho\sigma}\Big] \;=\; i\left(M_{\mu\sigma}\,\eta_{\nu\rho} \;+\; M_{\nu\rho}\,\eta_{\mu\sigma} \;-\; M_{\mu\rho}\,\eta_{\nu\sigma} \;-\; M_{\nu\sigma}\,\eta_{\mu\rho}\right).$$

Rotations J_i and Lorentz boosts K_i are defined by

$$J_i = \frac{1}{2}\,\epsilon_{ijk}\,M_{jk}, \qquad K_i = M_{0i}\ .$$

Show that the Lorentz algebra can be re-expressed in the form

$$[K_i,\ K_j] = i\epsilon_{ijk}(A_1 J_k + B_1 K_k), \qquad [J_i,\ K_j] = i\epsilon_{ijk}(A_2 J_k + B_2 K_k),$$

$$[J_i,\ J_j] = i\epsilon_{ijk}(A_3 J_k + B_3 K_k),$$

where A_1, A_2, A_3 and B_1, B_2, B_3 are integers, which you should determine.

(b) Discuss the hierarchy problem, the technical hierarchy problem and its supersymmetric solution. Include a discussion of vector boson, chiral fermion and squark/slepton masses as regards the hierarchy problem.

2

Write down the right hand side of the anti-commutator between the $N = 1$ global SUSY generators Q_α and $\bar{Q}_{\dot{\beta}}$

$$\{Q_\alpha,\ \bar{Q}_{\dot{\beta}}\}.$$

Next, using the fermion number operator $(-)^F$, calculate $\left\{(-)^F\ ,\ Q_\alpha\right\}$.

Hence prove that the number of bosons is identical to the number of fermions in a supersymmetric multiplet.

Considering now a supersymmetric theory that has supersymmetry breaking terms added to its Lagrangian, where does this argument fail?

Use $\left\{Q_\alpha,\ \bar{Q}_{\dot{\beta}}\right\}$ to derive conditions on the energy of the vacuum state $|\text{vac}\rangle$ for the cases of
(a) $N = 1$ global SUSY
(b) broken global SUSY.

Considering three chiral superfields Φ_+, Φ_0, Φ_- with gauged $U(1)$ charges +1, 0 and -1, respectively and superpotential $W = \lambda\Phi_+\Phi_0\Phi_-$, write down the resulting scalar potential in terms of the scalar components φ_+, φ_0, φ_-.

By minimising this potential determine whether (and how) the model spontaneously breaks:
(c) U(1)
(d) SUSY

3

Define what is meant by a chiral superfield Φ in terms of the superspace covariant deriative $\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^{\beta}(\sigma^{\mu})_{\beta\dot{\alpha}}\partial_{\mu}$. Derive the form of Φ in terms of component fields using the variables y^{μ}, θ^{α}, $\bar{\theta}^{\dot{\alpha}}$, where $y^{\mu} \equiv x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}$. Show that polynomials of Φ are also chiral superfields. Write down the Lagrangian in terms of a Kähler potential $K(\Phi^{\dagger}, \Phi)$ and the superpotential $W(\Phi)$.

Specialising to the Wess-Zumino model containing one single chiral superfield Φ, the tree-level superpotential is

$$W_{tree} = \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3.$$

Calculate the tree-level effective potential for the scalar component fields. Give the Feynman diagram and Feynman rule corresponding to the quartic scalar coupling.

Define two $U(1)$ symmetries of W_{tree}, where m and g are considered to be spurions. One of the $U(1)$s should be an $R-$symmetry with m being chargeless under it. Use this to derive the most general form of a loop-corrected effective superpotential $F(\Phi, m, g)$. Are W_{tree}, m or g renormalised? Give reasons for your answer.

END OF PAPER