UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 6 June, 2013 9:00 am to 11:00 am

PAPER 35

DESIGN OF EXPERIMENTS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

(a) Suppose that the experimental units are grouped into b blocks of size k. Denote by Y_ω the response on experimental unit ω, and by $\underline{Y}$ the column vector of all such responses. Assume that there are (unknown) constants τ_i (for treatment i), σ^2, ρ_1 and ρ_2 such that $E(Y_\omega) = \tau_i$ if treatment i is assigned to experimental unit ω, and that

$$\mathrm{Cov}(Y_\alpha, Y_\beta) = \begin{cases} \sigma^2 & \text{if } \alpha = \beta \\ \rho_1\sigma^2 & \text{if } \alpha \neq \beta \text{ but } \alpha \text{ and } \beta \text{ are in the same block} \\ \rho_2\sigma^2 & \text{otherwise.} \end{cases}$$

Find the eigenvalues and eigenspaces of the variance-covariance matrix $\mathrm{Cov}(\underline{Y})$, explaining any notation that you use.

(b) Define what it means for a block design to be *orthogonal*.

(c) A post-doctoral researcher in Biology is planning an experiment using 50 cells of *Escherichia coli*. She will add one extra green fluorescent protein to each of ten of the cells. Similarly, she will add two, three or four extra green fluorescent proteins to ten cells each, and leave the remaining ten cells with no added proteins. She will measure the rate of diffusion of proteins in each cell immediately after she has added the extra proteins, if any.

She will conduct this experiment in a single week, treating and measuring ten cells per day on each of Monday to Friday.

(i) How should this experiment be designed?

(ii) How should it be randomized?

(iii) Give the appropriate skeleton analysis-of-variance table, showing strata, sources and degrees of freedom.

2

A researcher in human-computer interaction wants to compare three computer programs (code-named A, B and C) which are intended to help people to draw sketch maps interactively. He recruits six volunteers from among Part III Mathematics students and asks them to participate on six Wednesday afternoons. Each afternoon, each volunteer will be asked to draw one sketch map using whichever program is assigned to him or her that afternoon. The researcher will record how many features of the map the volunteer managed to include correctly in the sketch.

(a) Identify the experimental units and observational units in this experiment, justifying your answer.

(b) Identify any suitable blocks for this experiment, giving reasons.

(c) Construct the design and randomize it, presenting the final plan in a form suitable for the researcher.

If you need them, you may use the following table of random numbers.

0.170, 0.615, 0.941, 0.075, 0.308, 0.975, 0.408, 0.118, 0.144, 0.085, 0.136, 0.301, 0.760, 0.809, 0.702, 0.948, 0.901, 0.652, 0.065, 0.146, 0.531, 0.574

(d) Suggest one other feature which might be considered in the design of this experiment, and explain why.

3

(a) Describe three different possible experimental designs for comparing two drugs (P and Q) on animals. For each design, briefly state how to construct and randomize it, and state one advantage and one disadvantage of the design.

(b) A vet has made an appointment with you to discuss her proposed trial of two new drugs P and Q for dairy cows, for a disease which you have never heard of. Make clear notes on two of the important points to discuss with her at your meeting. Include your reasons for discussing these points.

4

An experiment was conducted to find out the effects of pruning methods and chemical spraying on the growth of cider apples. Twelve orchards were used, because spray cannot realistically be applied to different nearby trees. In each orchard, 30 trees were used, all of the same variety.

The treatments consisted of all combinations of three different pruning methods with either spray or no spray. Each treatment was applied to two orchards. At harvest time, the weight of usable apples per tree was recorded.

(a) A simplified version of the data leads to the following analysis-of-variance table. Fill in all the blanks marked `xx`.

stratum	source	SS	df	MS	VR (to 1 sig. fig.)
mean	mean	1130412	1	1130412	
orchards	spray (S)	998	xx	xx	xx
	pruning (P)	1120	xx	xx	xx
	S-by-P	404	xx	xx	xx
	residual	1440	xx	xx	
	total	3962	11		
trees		62640	348	180	
Total		1197014	360		

(b) What conclusions can you draw about the presence or absence of treatment effects? Give reasons.

(c) Estimate the variance of the estimated difference between two pruning methods and the variance of the estimated difference between spray and no spray.

(d) State briefly what else should be given to complete the report of the analysis of these data.

(e) In general, if there are k trees per orchard, if the variance of the response on each tree is σ^2, the correlation between two trees in the same orchard is ρ, and trees in different orchards are uncorrelated, then the expectation of the residual means squares in the orchards and trees strata are $\sigma^2(1-\rho)+k\sigma^2\rho$ and $\sigma^2(1-\rho)$ respectively.

Estimate $\sigma^2(1-\rho)$ and $\sigma^2\rho$ in this experiment.

(f) A similar experiment will be conducted next year. Briefly compare the merits of the following possible modifications.

(i) Use 45 trees per orchard instead of 30.

(ii) Use 18 orchards instead of 12.

(iii) Apply each pruning method to ten individual trees per orchard.

END OF PAPER