UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 10 June, 2013 1:30 pm to 4:30 pm

PAPER 20

COMPUTABILITY AND LOGIC

Attempt no more than ***FOUR*** *questions.*

There are ***SIX*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

What is a *regular expression*? State and prove Kleene's theorem that the set of strings accepted by a deterministic finite state machine is captured by a regular expression.

What does it mean for a nondeterministic machine to accept a string? Is there a version of Kleene's theorem for nondeterministic finite state machines?

2

(i) Let $\mathfrak{M}$ be a machine that halts on all inputs. Is there a computable function f such that $f(n)$ bounds the time taken by $\mathfrak{M}$ to halt on input n? Suppose the function computed by $\mathfrak{M}$ is not total ... what then?

(ii) State and prove the Extended Omitting Types theorem for Propositional Logic.

3

Give a direct definition of the factorial function $\mathbb{N} \to \mathbb{N}$ in the language of ordered rings. Give an estimate of the length of your formula.

4

What is a typed λ-term? Explain the connection with constructive propositional logic. What is a Church numeral? Supply λ-terms for successor, addition, multiplication and exponentiation on Church numerals. What is the Y combinator? Sketch how it can be used to find a λ term for every computable function.

5

A *transversal* for a family $\mathcal{X}$ of pairwise disjoint subsets of a set X is a subset X' of X s.t. $|X' \cap x| = 1$ for all $x \in \mathcal{X}$.

Let $\sim$ be an equivalence relation on $\mathbb{N}$, with infinitely many equivalence classes, whose complement is semidecidable (considered as a subset of $\mathbb{N} \times \mathbb{N}$). Show that there is a semidecidable transversal on the set of $\sim$-equivalence classes.

6

State and prove Kruskal's theorem on wellquasiordering finite trees.

Suppose we quasiorder finite trees as follows: $T \leqslant T'$ if there is an injection from the vertex set of T to the vertex set of T' that preserves the root and preserves adjacency. Is this a WQO?

END OF PAPER