MATHEMATICAL TRIPOS   Part III

Friday, 31 May, 2013   1:30 pm to 4:30 pm

PAPER 2

COMMUTATIVE ALGEBRA

*Attempt no more than **FOUR** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

**You may not start to read the questions
printed on the subsequent pages until
instructed to do so by the Invigilator.**

1

Explain the theory of integral extensions far enough to show that, if A is a normal Noetherian domain and L is a finite seperable extension of the fraction field $Frac(A)$, then the integral closure of A in L is a ring that is finite over A.

[Standard results from Galois theory may be assumed.]

2

State and prove the Noether Normalization Lemma, and the weak and strong forms of the Nullstellensatz.

3

Write an essay on the primary decomposition of ideals in Noetherian rings and its relevance to the theory of Artinian rings.

4

Develop the theory of Kähler differentials far enough to relate it to the notion of transcendence basis.

5

Write an essay on Koszul complexes. Details about signs in tensor products of complexes do not require detailed proofs.

END OF PAPER