UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 7 June, 2012 9:00 am to 12:00 pm

PAPER 51

STRING THEORY

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

In closed bosonic string theory explain how, starting from the action, one constructs the four-point scattering amplitude for tachyons.

2

The action for a bosonic p-brane propagating in d-dimensional Minkowski spacetime is

$$I = \int_{\Sigma} d^{p+1}\xi \, \gamma^{1/2} (\gamma^{\mu\nu} \partial_\mu X^a \partial_\nu X^b \eta_{ab} - (p-1)) \, ,$$

where Σ is the world-volume of the p-brane, ξ^μ labels points on the world-volume, $X^a(\xi)$ are the spacetime co-ordinates of the point ξ^μ, η_{ab} is the Minkowski spacetime metric, $\gamma_{\mu\nu}$ is the world-volume metric and $\gamma = -\det \gamma_{\mu\nu}$.

Derive the equations of motion for X^a and $\gamma_{\mu\nu}$, taking care to explain how and why the case $p = 1$ differs from $p \neq 1$ and what boundary conditions are needed.

For the case $p = 1$, in other words the string, and assuming that we are dealing with the open string with NN boundary conditions, derive the operator version of the Virasoro constraints assuming we can choose the orthonormal gauge for the world-sheet metric.

Find the commutators of the Virasoro operator, paying particular attention to the central term.

3

The action for supersymmetric closed string theory in the orthonormal gauge is

$$I = \int_{\Sigma} d\sigma d\tau (\partial_\mu X^a \partial_\nu X^b \eta_{ab} \eta^{\mu\nu} + i\bar{\psi}^a \gamma^\mu \partial_\mu \psi^b \eta_{ab})$$

where τ and σ are time and space co-ordinates of the string world-sheet which has Minkowski metric $\eta_{\mu\nu}$, η_{ab} is the metric of the background Minkowski spacetime, X^a are the (bosonic) string-world-sheet coordinates, and ψ^a are a set of Majorana fermions which are the superpartners of X^a.

Explain what is meant by the terms Majorana fermion and chirality.

Show that it is possible to choose the positive chirality components to be right-moving on the string world-sheet and hence similarly that the negative chirality components are left-moving.

Show that rigid supersymmetry transformations

$$\delta X^a = i\bar{\epsilon}\psi^a\,, \quad \delta\psi^a = (\gamma^\mu \partial_\mu X^a)\epsilon\,,$$

where ϵ is a Majorana spinor, leave the action I invariant up to possible surface terms.

In the critical dimension for the left-moving sector only, demonstrate that the number of spacetime degrees of freedom for massless string states is the same for the bosonic excitations as the fermionic excitations, stating clearly any assumptions you need to make.

4

The action for the closed bosonic string in a curved spacetime with metric g_{ab} and with vanishing dilaton in the orthonormal gauge is

$$I = \int d^2\xi \quad \frac{1}{2}\partial_\mu X^a \partial_\nu X^b \eta^{\mu\nu} g_{ab}(X)\,,$$

where the world-sheet metric is flat $\eta_{\mu\nu}$, ξ^μ are the string world-sheet coordinates and X^a labels where in spacetime the points ξ^μ on the string are located.

Explain how to derive the background field equation

$$R_{ab} = 0\,,$$

where R_{ab} is the Ricci tensor of the metric g_{ab}, from requiring that the classical symmetries of the string are preserved quantum mechanically.

Suppose that the background metric has vanishing Ricci tensor and admits a (spacelike) Killing vector

$$k^a \frac{\partial}{\partial x^a} = \frac{\partial}{\partial z}$$

and that the d-dimensional background metric is of the form

$$g_{zz} = V, \quad g_{zI} = 0, \quad I = 1, 2, \ldots (d-1)\,,$$

with all metric coefficients independent of z. By making the appropriate field transformations on the string worldsheet, show that the T-dual metric with g_{zz} replaced by V^{-1} but g_{zI} and g_{IJ} unchanged is equivalent to the original string theory.

Briefly explain why the fact that $R_{ab} \neq 0$ for the new metric does not imply an inconsistency for bosonic string theory.

END OF PAPER