MATHEMATICAL TRIPOS Part III

Friday, 8 June, 2012  9:00 am to 11:00 am

PAPER 24

COMPUTABLE FUNCTION THEORY

Attempt no more than THREE questions.

There are FOUR questions in total.

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.
1

Exhibit a decidable partition of $[\mathbb{N}]^3$ (the set of triples from $\mathbb{N}$) that has no infinite decidable monochromatic set.

Exhibit a recursive perfect binary tree with no infinite recursive path.

2

Explain function-in-intension and function-in-extension.

What is a primitive recursive function? Is, for each $i$, the set of Gödel numbers of primitive recursive functions-in-intension of arity $i$ decidable? By diagonalising over an enumeration of such Gödel numbers, or otherwise, exhibit a computable total function that is not primitive recursive.

3

Prove that there is a semidecidable set $X \subseteq \mathbb{N}$ with $\mathbb{N} \setminus X$ infinite such that $X$ meets every infinite semidecidable set.

4

Explain many-one-reducibility and Turing-reducibility. Prove the theorem of Kleene and Post that there are incomparable Turing degrees below $0'$.

END OF PAPER