MATHEMATICAL TRIPOS Part III

Tuesday, 14 June, 2011 $\,$ 9:00 am to 12:00 pm $\,$

PAPER 76

SET THEORY

Attempt no more than **FOUR** questions. There are **SIX** questions in total. The questions carry equal weight.

STATIONERY REQUIREMENTS

Cover sheet Treasury Tag Script paper **SPECIAL REQUIREMENTS** None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

UNIVERSITY OF

1

State and prove Fodor's theorem on regressive functions and stationary sets.

 $\mathbf{2}$

$\mathbf{2}$

State and prove Solovay's theorem on partitioning stationary sets into many stationary sets.

3

State and prove Silver's theorem on powers of singular cardinals.

$\mathbf{4}$

State and prove the theorem of Erdős and Rado on the existence of monochromatic sets for partitions of sets of unordered k-tuples from uncountable sets.

$\mathbf{5}$

What is a normal measure?

Let κ be a measurable cardinal, D a normal measure on κ , and F a partition of $[\kappa]^{<\omega}$ into fewer than κ pieces. Show that there is a set $H \in D$ that is monochromatic for F.

Explain why this implies that every measurable cardinal is Ramsey.

6

State and prove the Reflection Principle.

END OF PAPER