UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 6 June, 2011 1:30 pm to 4:30 pm

PAPER 74

SOFT MATTER AND BIOLOGICAL PHYSICS

*Attempt **ALL** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider a dielectric, spherical particle with radius R (of order of 1 micron), mass m and friction coefficient β immersed in water, which is held in an optical trap. Which are the main forces on the particle in an optical trap?

The trapping potential can be approximated as harmonic. The potential is characterised by a trap stiffness κ and corresponding frequency f_c. Write down the Langevin equation in one dimension for this particle taking into account the thermal random force ($\mathfrak{F}$). Under which conditions can one neglect the inertial acceleration term in the Langevin equation?

With the the constant power spectrum of an ideal white noise source $S_{\mathfrak{F}} = |\mathfrak{F}(f)|^2 = 4\beta k_B T$ calculate the power spectrum of the particle motion in the optical trap $S_x(f) = |x(f)|^2$. Sketch $S_x(f)$ for several values of κ and label all axis and relevant points?

What happens for frequencies $f > f_c$ with the power spectrum when the laser power is increased? Remember that part of the laser power is absorbed in the solution. Briefly, describe two techniques to register the motion of the particle in the trap.

2

Consider a capillary tube with radius r much larger than the Debye-Huckel screening length λ containing monovalent dissociated salt ions of concentration $c_0 = c_0^+ = c_0^-$ and resistivity ρ. The capillary has length l and connects two semi-infinite reservoirs. Calculate the total resistance of the capillary taking into account the access resistance given by $\rho/4r$. Derive the total resistance for a conical capillary with different radii at both its ends r_1 and r_2, where $r_1 > r_2$. Sketch the electric field and potential for the cylindrical case.

Assume that the surface of the cylindrical capillary is charged and has a fixed surface potential $\zeta < 0$. Under the assumption that $r \gg \lambda$ show that the fluid velocity v of the electro-osmotic flow in the centre of the capillary can be written as

$$v = -\frac{\epsilon_0 \epsilon_r \zeta E}{\theta}$$

where θ is the fluid viscosity and E the applied electric field along the capillary. Sketch the velocity as a function of r.

A very long charged polymer with contour length $L \gg l$, surface potential $\zeta_p < 0$ and corresponding charge q per Kuhn length is placed in front of one of the capillary openings. Calculate the change in potential energy of the polymer when it enters the capillary assuming that its Kuhn segment length is much larger than r. Sketch the potential energy along the capillary. What is the condition for the polymer to enter the capillary?

3

Consider a charged surface in aqueous solution with surface potential ϕ_0. The surface is immersed in a solvent containing monovalent dissociated salt ions of concentration $c_0 = c_0^+ = c_0^-$. Solve the Poisson-Boltzmann equation for the electrostatic potential $\phi(x)$ in this geometry and thus derive the formula for the Debye-Huckel screening length λ in the limit of small surface potential $e\phi_0 \ll k_B T$. Discuss the dependence of the screening length on c_0. Sketch and discuss the distribution of the positive and negative ions close to the surface for $e\phi_0 \ll k_B T$ and $e\phi_0 > k_B T$ and briefly comment on the difference.

A second, identical surface is now held in close proximity to the first surface at a distance D. Find a solution for the Poisson-Boltzmann equation again in the limit of $e\phi_0 \ll k_B T$. Make a sketch of the potential between the surfaces.

What happens to $\phi(x = 0)$ when (i) $D \to \infty$ or (ii) $D \to 0$?

By calculating the total electrostatic energy per unit area of this ionic solution between the surfaces, $E(D)$, find the added pressure between them and sketch it as a function of D.

END OF PAPER