UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 9 June, 2011 9:00 am to 12:00 pm

PAPER 44

STRING THEORY

Attempt no more than ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS

Cover sheet

Treasury Tag

Script paper

SPECIAL REQUIREMENTS

None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

What is the Nambu-Goto action for the closed bosonic string?

Describe how this is a generalization of the action for a point particle.

Construct the Polyakov action for the closed bosonic string in d-dimensional Minkowski spacetime starting from the Nambu-Goto action.

Describe all of the symmetries of this theory.

Describe carefully how one shows that closed bosonic string theory is a theory containing gravitons if $d = 26$.

2

The amplitude for the scattering of four tachyons in closed bosonic string theory is

$$A = Cg_s^2\delta(\text{momentum conservation})\times \frac{\Gamma(-1-\alpha's/4)\Gamma(-1-\alpha't/4)\Gamma(-1-\alpha'u/4)}{\Gamma(2+\alpha's/4)\Gamma(2+\alpha't/4)\Gamma(2+\alpha'u/4)}$$

where α' is the inverse string tension, g_s is the string coupling constant, s, t and u are the Mandelstam variables for the scattering process and C is some normalization constant. Sketch a derivation of this formula starting from the Polyakov action.

[*You are not required to give detailed derivations, nor to evaluate complicated integrals, but to summarize and explain the steps leading to this result.*]

The amplitude contains a set of poles when viewed as a function of any of s, t or u. Explain the physical meaning of these poles.

In what ways does this result differ from four-particle scattering amplitudes found in the quantum field theory of scalars with a cubic coupling?

3

Explain what is meant by the terms quasi-primary operator of weight $(h, \bar{h})$, and primary operator in a conformal field theory.

Two anti-commuting operators are b and c and their equation of motion is $\bar{\partial} b = \bar{\partial} c = 0$. Their operator product expansion is given by

$$b(z)c(w) = -c(w)b(z) = \frac{1}{z-w} + \dots$$

with no singular terms in the expansion of either $b(z)b(w)$ or $c(z)c(w)$. Consider the stress tensor

$$T = :(\partial b)c: - \lambda \partial :bc: \quad , \quad \bar{T} = 0.$$

where λ is real. Show that b and c are primary operators and find h and $\bar{h}$ for both fields b and c. What is the central charge of this system?

4

Consider a single free spin-1/2 fermion ψ on the world sheet of a closed string in Minkowski spacetime. The action for such a field is

$$\int d^2x \bar{\psi}\, \gamma^\mu \partial_\mu \psi$$

Develop the mode expansions for both the NS and R sectors of the left-movers for this field in the lightcone gauge. Find expressions for the Virasoro operators in both R and NS sectors.

Show that the Virasoro algebra has commutation relations

$$[L_m, L_n] = (m-n)L_{m+n} + C(n)\delta_{n+m,\,0}\,.$$

Show that $C(n) = -C(-n)$.

By using the Jacobi identity, or otherwise, show that $C(n) = c_3 n^3 + c_1 n$.

Evaluate c_1 and c_3 for both the NS and R sectors.

END OF PAPER