UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 10 June, 2011 9:00 am to 11:00 am

PAPER 40

SUPERSYMMETRY

Attempt no more than ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

Please use the following conventions

$$\epsilon^{12} = -\epsilon_{12} = \epsilon^{\dot{1}\dot{2}} = -\epsilon_{\dot{1}\dot{2}} = -1, \qquad \epsilon_{0123} = +1 = -\epsilon^{0123}$$

$$(\theta\theta) \equiv \theta^\alpha \theta_\alpha, \qquad (\bar{\theta}\bar{\theta}) \equiv \theta_{\dot{\alpha}} \theta^{\dot{\alpha}},$$

The D-term scalar potential for a gauge group a is

$$V_D = \frac{g_a^2}{2} D^b D^b, \tag{1}$$

where D^b is is the auxiliary field in the vector superfield of gauge group a.

In $M_{pl} = 1$ units, the general $N = 1$ supergravity scalar potential is given by

$$V = e^K [K_{ij}^{-1} D_i W D_j W^* - 3|W|^2],$$

where $D_i W = \partial_i W + W \partial_i K$.

The Pauli matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and $\sigma^\mu = (1,\ \underline{\sigma})$.

The generators of the Poincaré group satisfy

$$[M^{\mu\nu},\ P^\sigma] = i(P^\mu \eta^{\nu\sigma} - P^\nu \eta^{\mu\sigma}).$$

Under a Lorentz transformation, a left-handed spinor ψ transforms as
$\psi \to \exp(\frac{-i}{2} w_{\mu\nu} \sigma^{\mu\nu})\psi$, where

$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$$

satisfies a self-duality condition

$$\sigma^{\mu\nu} = \frac{1}{2i} \epsilon^{\mu\nu\rho\sigma} \sigma_{\rho\sigma}.$$

The Jacobi identity for two bosonic operators A,B and one fermionic operator C is

$$0 = [A,\ [B,\ C]] + [B,\ [C,\ A]] + [C,\ [A,\ B]].$$

1

The MSSM contains two Higgs doublets. What are their representations under the Standard Model gauge group SU(3)×SU(2)×U(1)? State the two most important reasons for why there is an additional Higgs doublet as compared to the Standard Model.

Show that the $D-$terms of the MSSM Higgs potential are:

$$V_D = F(g,g')(|H_2^0|^2 + |H_2^+|^2 - |H_1^0|^2 - |H_1^-|^2)^2 + G(g,g')|H_2^+ H_1^{0*} + H_2^0 H_1^{-*}|^2$$

where, in doing so, you should determine the functions F and G of the SU(2) and U(1) gauge couplings g and g', respectively.

The MSSM Higgs SUSY breaking terms are

$$V_{SB} = B\mu(H_2^+ H_1^- - H_2^{0*} H_1^0) + H.c. + m_{H_1}^2(|H_1^0|^2 + |H_1^-|^2) + m_{H_2}^2(|H_2^0|^2 + |H_2^+|^2).$$

State any other contributions to the MSSM Higgs potential not contained within V_{SB} or V_D.

Calculate the mass squared eigenvalues of the neutral Higgs fields. Imposing that one of the eigenvalues be negative, as required by the Higgs mechanism, derive an inequality for $B\mu$ in terms of the other Higgs potential parameters.

2

The Polonyi model of $N = 1$ supergravity has a single chiral superfield z, a superpotential

$$W = m^2(z + \beta),$$

where β is a real constant, and Kähler potential

$$K = |z|^2.$$

(a) Calculate whether the model breaks or preseves supersymmetry.
(b) Calculate the scalar potential of z, $V_F(z)$.
(c) Imposing the observational constraint of a zero cosmological constant $V_F(\langle z \rangle) = 0$, where $\langle z \rangle$ is the vacuum expectation value of the scalar superfield, show that

$$\beta = \sqrt{A} + B,$$

where A and B are integers that you should find. You may assume that $\beta > 0$.
(d) Show that

$$\langle z \rangle = \sqrt{C} + D,$$

in Planck units, where C and D are integers which you should find.

3

Write down the equations of the $N = 1$ global supersymmetry algebra with the following quantities on the left-hand side:
(a) $[Q_\alpha,\ M^{\mu\nu}]$,
(b) $[\bar{Q}^{\dot{\alpha}},\ M^{\mu\nu}]$,
(c) $[\bar{Q}^{\dot{\alpha}},\ P^\mu]$,
(d) $\{Q_\alpha,\ Q_\beta\}$,
(e) $\{Q_\alpha,\ \bar{Q}_{\dot{\beta}}\}$.

Consider the superspin operator

$$\tilde{C}_2 = C_{\mu\nu} C^{\mu\nu}$$

where $C_{\mu\nu} = B_\mu P_\nu - B_\nu P_\mu$, $B_\mu = W_\mu - \frac{1}{4}(\bar{Q}\bar{\sigma}_\mu Q)$ and $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$. How does one expect $\tilde{C}_2$ to transform under the Lorentz group? What does that mean for its commutator with $M^{\mu\nu}$?

Calculate:
(f) $[B_\mu, P_\rho]$
(g) $[C_{\mu\nu}, P_\rho]$
(h) $[\bar{Q}\bar{\sigma}_\mu Q,\ Q_\alpha]$
(i) $[B_\mu,\ Q_\alpha]$
(j) $[C_{\mu\nu},\ Q_\alpha]$
and so demonstrate that $\tilde{C}_2$ is a Casimir of the super-Poincaré algebra.

END OF PAPER