UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 13 June, 2011 9:00 am to 12:00 pm

PAPER 4

TOPICS IN REPRESENTATION THEORY

*Attempt no more than **FOUR** questions.*

*There are **SIX** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 How many simple kG-modules are there up to isomorphism when $G = SL(2, p)$ and k is an algebraically closed field of characteristic p. Briefly justify your answer. Describe them, proving that they are simple.

2 Define what is meant by a block B of the group algebra kG when k is a field of characteristic p and G is a finite group. Define the defect group of B and show that it is isomorphic to a p-subgroup of G.

Let the characteristic of k be 2. For kS_3 describe the simple and the indecomposable projective modules. What are the blocks, and their defect groups?

3 Let k be a field and G be a finite group.

Define what is meant by a vertex of an indecomposable left kG module.

Show that if B is a block with defect group D then any indecomposable left kG-module lying in B has vertex contained in D.

4 Let k be a field of characteristic p.

Show that if a group G has order p^a then the only simple kG-module is the trivial one.

Let N be a normal subgroup of a finite group H with the order of H/N equal to p. Show that if V is an indecomposable left kN-module then the kH-module induced from V is also indecomposable.

5 Let k be a field and A be a k-algebra.

Define the Hochschild cohomology groups $HH^n(A, A)$ and define the dimension $Dim(A)$ of A.

What does it mean for A to be a separable k algebra? Show that A is a separable k-algebra if and only if $Dim(A) = 0$. Show that if this is the case then A is a finite dimensional k vector space and is semisimple.

Give an example of a k-algebra A with $Dim(A) = 1$. Justify your answer.

6 Write an essay about the relationship between the Hochschild cohomology of a k-algebra and its deformation theory.

END OF PAPER