UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 9 June, 2011 1:30 pm to 3:30 pm

PAPER 22

COMPUTABLE FUNCTION THEORY

*Attempt no more than **THREE** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

What is the Ackermann function? Prove that it dominates all primitive recursive functions and is therefore not primitive recursive.

2

Explain *function-in-intension* and *function-in-extension*. Prove that it cannot in general be decided when two functions-in-intension have the same extension. If you wish to appeal to a general result you should prove it.

3

Establish that a function $\mathbb{N}^k \rightharpoonup \mathbb{N}$ is λ-representable iff it is μ-recursive.

4

Prove that the set of (indices of) total computable functions is not semidecidable. Explain how the formalisation of this in an axiomatic theory of arithmetic can prove Gödel's theorem on undecidability. What is a productive set of natural numbers? Give an example.

5

Explain *many-one* reducibility and *Turing-reducibilty*. Prove the theorem of Kleene and Post that there are sets neither of which is Turing-reducible to the other.

END OF PAPER