UNIVERSITY OF
CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 3 June, 2010 1:30 pm to 4:30 pm

PAPER 60

GALAXIES

Candidates may bring handwritten class notes only into the examination.

Attempt no more than ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider a self-gravitating disc of stars with vertical velocity dispersion σ, which for simplicity can be considered to be constant.

(i) Show that the vertical distribution of stellar density falls off exponentially, with an e-folding scale height:

$$h = \frac{\sigma^2}{2\,\pi G \Sigma}\,,$$

where Σ is the mass surface density of the disc (the volume density integrated vertically through the disc).

(ii) The gas discs of spiral galaxies are observed to have a vertical velocity dispersion that is virtually constant with radius. Derive an expression for the expected radial dependence of the gas scale height, assuming that the equation above applies to the gas (but with the surface density Σ dominated by the stars). How does this compare to the radial dependence of h for the stellar discs?

(iii) Observations of edge-on galaxies show that the radial behaviour of the gas scale height predicted above applies over a limited range of radii, but breaks down badly at other radii. When will the assumptions underlying the derivation of the equation in part (i) break down for this problem?

(iv) Describe how measurements of the vertical distribution and velocity dispersion of stars in the Galactic disc could be used to place constraints on the fraction of dark matter in the disc.

2

In the lectures we considered the chemical evolution of a closed system of stars and gas, in which gas was not introduced into or ejected from the system. Consider instead a system in which metal-free gas accretes on to the galaxy at a rate that is equal to a fraction ν of the star formation rate. For simplicity you may ignore any internal recycling of gas from stellar evolution (i.e., set the recycling term $R = 0$).

(i) Show that the metal abundance of the gas $Z(t)$ can be expressed as a function of the ratio of the gas mass $M(t)$ to the initial gas mass $M(0)$ as

$$Z(t) = \frac{y}{\nu}\left[1 - \left(\frac{M(t)}{M(0)}\right)^{\nu/(1-\nu)}\right] ,$$

where y is the heavy element yield.

Hint: Note that the gas mass ratio above is not the same as the gas fraction discussed in class and in the examples sheet, so don't expect the same approach you applied there to necessarily work here.

(ii) For the simple case of $\nu = 0.5$ (which is realistic for the Milky Way disc) derive an expression for the number of stars as a function of metal abundance $N(Z)$.

(iii) Plot a (schematic) graph which compares the metallicity distribution you derived above with the $N(Z)$ distribution for the closed box model. Explain qualitatively how the steady introduction of metal-free infalling gas accounts for the differences in the metal abundance distributions.

(iv) The simple closed box model is infamous for failing to reproduce the observed distribution of metal abundances of stars in the Galactic disc (G-dwarf problem). In what respects does the simple infall model above alleviate these discrepancies, and in which respects does it still fail?

3

Eggen, Lynden-Bell, and Sandage (ELS) are famous for developing a model for the formation of the Galaxy in which most of the present day material collapsed on timescales comparable to a free-fall time.

(i) If we presume for simplicity that the orbital velocities of stars in the galaxy are constant with radius and isotropic (i.e., a spherical system), derive an expression for the dynamical time in terms of the total mass of the system (M) and its velocity dispersion (σ).

(ii) Use this result to estimate the dynamical time for a spheroidal galaxy with gross properties similar to that of the Milky Way, i.e., for an assumed mass of $10^{11}\mathrm{M}_\odot$ and velocity dispersion of 200 km/sec. Since calculators are not available it is fine to approximate to one significant figure (don't worry about decimal accuracy). Is this number sensible based on what you know about the age and orbital timescale for the Milky Way?

(iii) Suppose, as postulated in the extreme form of the ELS picture, that all of the mass above were in the form of gas, and it formed into stars over a dynamical time. What would be the primordial star formation rate over this phase? How does it compare to the present-day star formation rate of the Milky Way?

(iv) We know from the virial theorem that for the Galaxy to collapse over a dynamical time a fraction of its total energy must be radiated away from the system. Estimate this energy, and the average luminosity that would be emitted over this dynamical time. How does it compare to the present-day luminosity of typical L$*$ galaxies? (Of course this starlight arises from completely different physical processes!)

(v) Observations show that most if not all massive spheroidal galaxies contain a central black hole, with a mass of order 1/1000 that of the galaxy itself. Suppose that this black hole also had to form coevally with the galaxy itself, over the same (galactic) dynamical time. How much energy would need to be released to form the black hole, and what would the average luminosity be over this period? How does that luminosity compare to the present-day luminosities of L$*$ galaxies?

Useful Information:

$$G = 7 \times 10^{-8}\mathrm{cm}^3\mathrm{g}^{-1}\mathrm{s}^{-2} = 7 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$$

$$\mathrm{M}_\odot = 2 \times 10^{33}\mathrm{g} = 2 \times 10^{30}\mathrm{kg}$$

$$1\ \mathrm{pc} = 3 \times 10^{18}\mathrm{cm} = 3 \times 10^{16}\mathrm{m}$$

$$1\ \mathrm{yr} = 3 \times 10^{7}\ \mathrm{s}$$

$$c = 3 \times 10^{10}\mathrm{cm\,s}^{-1} = 3 \times 10^{8}\mathrm{m\,s}^{-1}$$

4

Mini essay questions: Explain in no more than two or three sentences each the astrophysical explanations for each of these observations of galaxies and stellar populations. You may include equations if you wish but it is not required.

(i) Old stars in the Galactic disc at a given radius lag in rotation behind younger stars.

(ii) A census of the low-mass stellar populations within a few tens of parsecs of the Sun provides a fair sampling of the Galactic disc at 7 kpc radius, even though the volume samples only a tiny fraction of the disc.

(iii) A galaxy with weak metal absorption lines in its spectrum is not necessarily more metal-poor than a galaxy with strong metal lines in its spectrum.

(iv) Present-day elliptical galaxies have a significantly higher mass-to-light ratio than spiral or irregular galaxies.

(v) The age distribution of open clusters in the Milky Way is strongly peaked toward ages of less than a Gyr, quite distinct from either the globular clusters in the Galactic halo or the field stars in the Galactic disc.

(vi) Massive elliptical galaxies tend to have boxy $\cos 4\theta$ components to their azimuthal luminosity profiles, in the opposite sense to the $\cos 4\theta$ components in low-mass elliptical galaxies.

(vii) The strongest constraints on the distribution of dark matter in galaxies comes from studies of atomic hydrogen, rather than from kinematics of stars.

(viii) Measurements of the masses of black holes in the centres of elliptical galaxies are much easier to obtain for low-mass galaxies, even though the masses of the central black holes are much lower.

END OF PAPER