UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 8 June, 2010 1:30 pm to 3:30 pm

PAPER 56

SUPERGRAVITY

Attempt no more than ***THREE*** *questions.*

There are ***FIVE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Explain what is meant, in four spacetime dimensions, by a Majorana spinor and by a Weyl spinor. Which of the two can satisfy a massive Dirac equation and which must satisfy a massless Dirac equation? Which is its own anti-particle and why?

A Rarita-Schwinger field ψ_σ in four dimensional Minkowski spacetime satisfies

$$\gamma^{\mu\nu\sigma}\partial_\mu \psi_\sigma = 0 .$$

Show that the field is massless and carries two physical degrees of freedom.

How can one add a mass term and what difference does it make to the degree of freedom count?

2

If ψ is a Dirac spinor and ∇ denotes the connection induced from the Levi-Civita connection, show that the Dirac equation

$$\gamma^\mu \nabla_\mu \psi + m \psi = 0$$

leads to the equation

$$-\nabla^2\psi + \frac{R}{4}\psi + m^2\psi = 0 .$$

If $\{M, g\}$ is a compact Riemannian manifold (i.e with positive definite metric g) without boundary whose Ricci scalar is non-negative, show that every non-singular non-vanishing solution of

$$\gamma^\mu \nabla_\mu \psi = 0$$

must be covariantly constant. By considering the Ricci identity and contracting with a suitable γ matrix, show that if there is a non-trivial solution then the Ricci tensor must vanish.

3

Write down the simple supergravity action in four spactime dimensions, ignoring 4-fermi terms.

Show that your action is invariant up to boundary terms under

$$\delta e_\mu^{\,a} \;=\; 2\,\kappa\,\bar{\epsilon}\,\gamma^a\,\psi_\mu\,, \qquad \delta\psi \;=\; \frac{1}{\kappa}\,\nabla_\mu\,\epsilon\,.$$

Again ignoring 4-Fermi terms, show what modifications are required to incorporate:

- a cosmological constant term.
- the electromagnetic field.

4

Explain what is meant by a Killing spinor ϵ. Show that for simple supergravity, if ϵ is taken to be commuting, then $K^\mu = \bar{\epsilon}\,\gamma^\mu\epsilon$ is a Killing vector field. Show that K^μ is nowhere spacelike.

Give a brief account of the supergravity proof of the positive energy theorem starting from the Nester two-form, and the part played in the argument by Killing spinors.

5

Write an essay on the derivation of the Einstein equations in the presence of matter, both bosonic and spinorial, from suitable variational principles. Your answer should cover both first order and second order formulations and explain what is meant by the 1.5-order fomalism.

END OF PAPER