UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 4 June, 2010 9:00 am to 11:00 pm

PAPER 40

SUPERSYMMETRY

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

Please use the following conventions

$$\epsilon^{12} = -\epsilon_{12} = \epsilon^{\dot{1}\dot{2}} = -\epsilon_{\dot{1}\dot{2}} = -1\,,$$

$$(\theta\theta) \equiv \theta^{\alpha}\,\theta_{\alpha}\,, \qquad (\bar{\theta}\bar{\theta}) \equiv \theta_{\dot{\alpha}}\,\theta^{\dot{\alpha}}\,,$$

$$D_{\alpha} = \left[\frac{\partial}{\partial\theta} + i\sigma^{\mu}\bar{\theta}\partial_{\mu}\right]_{\alpha}\,, \qquad \bar{D}_{\dot{\alpha}} = \left[-\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^{\mu}\partial_{\mu}\right]_{\dot{\alpha}}\,,$$

$$\mathcal{H} = (\bar{\sigma}^{0})^{\dot{\beta}\alpha}\left\{Q^{A}_{\alpha} - \Gamma^{A}_{\alpha},\, \bar{Q}_{\dot{\beta}A} - \bar{\Gamma}_{\dot{\beta}A}\right\}\,,$$

where $\Gamma^{A}_{\alpha} = \epsilon_{\alpha\beta}U^{AB}\,\bar{Q}^{B}_{\dot{\gamma}}(\bar{\sigma}^{0})^{\dot{\gamma}\beta}$ *and the complex matrix* U^{AB} *is unitary.*
$\sigma^{\mu} = (1, \underline{\sigma})$, *where* $\underline{\sigma}$ *are the Pauli matrices.*

In natural units, the conversion constant between seconds and inverse width in GeV is $1s \sim \mathcal{O}(10^{24})$ GeV^{-1} *and the proton mass is* ~ 1 *GeV.*

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Write a table detailing the entire propagating field content (including matter and gauge particles) of the Minimal Supersymmetric Standard Model prior to electroweak symmetry breaking, showing the representation of each component field under $SU(3) \times SU(2) \times U(1)_Y$, stating the spin of each field and the number of families. Mark the fields with additional anti-particle field degrees of freedom with a superscript $^{(A)}$.

Provide a definition of R-parity and then, in terms of superfields, write down the part of the MSSM superpotential that is consistent with R-parity. State the impact of R-parity conservation for collider sparticle searches, dark matter and the lifetime of the proton.

Write down the R-parity violating superpotential of the MSSM in terms of superfields. Draw a Feynman diagram representing proton decay. Assuming that the dimensionless R-parity violating couplings are of order 1, roughly estimate the proton lifetime τ in terms of the mass of the proton m_p and the relevant sparticle mass m. Given that $\tau > 10^{40}$ seconds from experiment and $m_p \sim 1$ GeV, provide an order of magnitude bound on m in units of GeV.

Is this bound on m expected to be satisfied in the MSSM? Give reasons for your answer.

2

Write an essay on BPS states, including a derivation of the BPS bound, the existence of short multiplets and applications of BPS states.

3

Write down the condition on a superfield Φ such that it is chiral. Derive its form $\Phi(y^\mu,\, \theta^\alpha,\, \bar{\theta}^{\dot{\alpha}})$ in terms of component fields evaluated at y^μ, where $y^\mu \equiv x^\mu + i\theta\sigma^\mu\bar{\theta}$. Derive constraints on the functional form of a function $f(\Phi)$ such that $f(\Phi)$ is a chiral superfield. Write down the Lagrangian in terms of the Kähler potential K and the superpotential W.

Specialising to the Wess-Zumino model containing one single chiral superfield Φ, the tree-level superpotential is

$$W_{tree} \,=\, \frac{1}{2}\, m\, \Phi^2 + \frac{1}{3}\, g\, \Phi^3\,.$$

Calculate the tree-level effective potential. Give the Feynman diagrams and Feynman rules corresponding to the tri-linear scalar couplings.

Define two $U(1)$ symmetries of W_{tree}, where m and g are considered to be spurions. One of the $U(1)$s should be an $R-$symmetry with m being chargeless under it. Thus derive the most general form of a loop-corrected effective superpotential $F(\Phi, m, g)$. Are W_{tree}, m or g renormalised? Give reasons for your answer.

END OF PAPER