UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 7 June, 2010 1:30 pm to 3:30 pm

PAPER 11

ADDITIVE COMBINATORICS

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

State and prove Roth's theorem on arithmetic progressions of length 3, using discrete Fourier analysis. (You may assume basic facts about the discrete Fourier transform, such as Parseval's identity, but other intermediate results should be proved.)

2

(i) State and prove Szemerédi's regularity lemma.

(ii) Let X and Y be sets of cardinality n, and let $f : X \times Y \to \mathbb{C}$. Define the U^2 norm of f. Show that

$$|\,\mathbb{E}_{x\in X}\,\mathbb{E}_{y\in Y}\,f(x,y)\,u(x)\,v(y)| \leqslant \|f\|_{U^2}\,\|u\|_2\,\|v\|_2$$

for any functions $u : X \to \mathbb{C}$ and $v : Y \to \mathbb{C}$.

3

Proving any facts that you might need concerning the sizes of sumsets, show that for every C there exists K with the following property: if N is a positive integer and A is a subset of $\mathbb{F}_2^N$ such that $|A+A| \leqslant C|A|$, then there is a subspace V of $\mathbb{F}_2^N$ such that $A \subset V$ and $|V| \leqslant K|A|$.

END OF PAPER