UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 5 June, 2009 9:00 am to 12:00

PAPER 80

GEOLOGICAL FLUID MECHANICS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

*This is an **OPEN BOOK** examination.*
Candidates may bring handwritten notes and lecture handouts into the examination.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Consider a two-dimensional, rectangular lava lake, open to the atmosphere, of horizontal length L and vertical height $H \equiv \epsilon L$ filled with Newtonian lava of constant kinematic viscosity ν and thermal conductivity κ. The free upper surface of the liquid lava is maintained with a constant horizontal temperature gradient $\Delta T/L$, while the sides and bottom are insulated.

Nondimensionalizing lengths, time and temperature differences with respect to $L, L^2/\kappa$ and ΔT respectively and using axes with x horizontal $(0 \leqslant x \leqslant 1)$ and z vertical $(0 \leqslant z \leqslant \epsilon)$, with $z = 0$ at the lava surface, write down the full nonlinear equations and boundary conditions satisfied by the nondimensional streamfunction $\psi(x, z, t)$ and temperature $T(x, z, t)$, defining clearly any parameters that appear.

Simplify this differential system under the assumption that the situation is steady and the Prandtl number is infinite.

With these assumptions, determine the lowest order solution for T and the consequent ψ when $\epsilon \ll 1$.

Evaluate the horizontal heat transport corresponding to this solution. Where does the heat come from? Where does the heat go to? What is the vertical heat transport?

Determine the correction to the temperature profile in response to the lowest order ψ solution you obtained.

2 Write down equations governing the radius, vertical velocity and reduced gravity for a light Boussinesq turbulent plume rising in a stratified ambient. State clearly the assumptions under which these equations are valid.

Find analytical solutions of these equations when the square of the buoyancy frequency of the ambient fluid $N^2 = Cz^p$, where z is the vertical co-ordinate with $z = 0$ at the plume source, and where C and p are constants (not necessarily positive).

Evaluate the specific mass, momentum and buoyancy fluxes for your solution.

Examine the validity and physical significance of the solution for different values of C and p.

3 A dam full of water of volume W suddenly bursts at the head of a V-shaped valley. Evaluate how the length of the resultant flow depends on time t and volume W. Explain clearly any assumption you make.

In a neighbouring, similar V-shaped valley, a short-lived pyroclastic flow of small heavy particles is initiated. Examine now how the flow depends on the volume of the eruption W and time t. Evaluate an expression for the final length of the flow, clearly defining every symbol you introduce. Calculate the density of the deposit as a function of downstream distance after the eruption has ceased.

4 Consider a solid spherical particle of freezing temperature T_s and initial radius a_0 sitting in an infinite supercooled liquid domain initially at temperature T_∞.

Write down the differential equation for the temperature $T(r,t)$ and the appropriate boundary conditions <u>including</u> the Gibbs-Thompson effect.

Define the Stefan number, and explain its physical significance.

By scaling the governing equations or otherwise, simplify your differential system appropriately for large Stefan number and show that there is a critical initial radius that separates growth of the initial particle from decay. By considering very small particle radii or otherwise, prove that a particle of initial radius less than the critical disappears in finite time.

END OF PAPER