UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday, 3 June, 2009 1:30 pm to 3:30 pm

PAPER 77

THE POLAR OCEANS AND CLIMATE CHANGE

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

In all questions assume that the latent heat of fusion of ice is 336 kJ kg^{-1}, and its specific heat is 2.1 kJ kg^{-1} K^{-1}.

1 Derive the equation for the dispersion relation of flexural-gravity waves in a uniform ice sheet, assumed to be perfectly elastic. The equation should express the wave number in ice as a function of thickness h, radian frequency ω, ice and water densities, Young's modulus and Poisson's ratio. Sketch the resulting general shape of the curve of group velocity versus wave period and explain the implications for wave generation in ice by wind. How can this model be modified to provide a simple theory for the dispersion relation of waves in pancake or frazil ice?

2 Give a general description of the structure of convective chimneys as observed in the Greenland Sea. What are the factors in the regional ocean and atmospheric structure which lead to the generation of chimneys, and why have they become more unfavourable in recent years? Explain the salt flux mechanism for enhancing overturning in the chimney region.

Assuming that the uppermost 100 m of the ocean participates in surface processes, calculate the heat lost from the ocean surface and the increase in density of surface water when this water column is cooled from 0°C to -1.8°C (freezing point). Calculate the subsequent increase in density of the surface water when the same amount of heat is extracted from the surface at the freezing point to grow ice (assume complete loss of salt within the ice). What does this tell us about the relative efficacy of cooling and freezing in convection? (Assume sea water density is 1025 kg m^{-3} , and that at low temperatures a density increase of 1 kg m^{-3} corresponds to a cooling of 10°C or a salinity increase of 1 psu.)

3 Several mechanisms have been suggested as contributing to the recent retreat of Arctic sea ice in summer. Describe the mode of operation and likely relative contributions of (i) air temperature changes, (ii) increased heat in the Atlantic layer, (iii) erosion of the cold halocline, (iv) Pacific water inflow, (v) changes in the phase of the Arctic Oscillation, (vi) long-term thinning, (vii) albedo changes. Explain how these mechanisms interact, and describe the likely future course of ice retreat in summer and winter during coming decades.

An approximate relation (due to Lebedev) for ice growth from open water during winter is $H = 0.0133N^{0.58}$, where N is number of degree-days of cold and H is thickness in m. If the present-day thickness at the end of a winter of duration 275 days is 1.5 m, estimate the effect on this thickness of

(a) an average air temperature rise of 4°C,

(b) an increased oceanic heat flux of 6 W m^{-2}.

4 Describe the role of air stress, water stress and Coriolis force in determining the motion of icebergs. Describe the transient and the equilibrium free drift motion of an iceberg acted on by the wind in still water. In the Antarctic, explain how the driving forces determine the trajectories of icebergs relative to ocean current systems and water masses. A tabular iceberg 1 km square and 300 m thick has a uniform density gradient in its upper 50 m, changing from 300 kg m^{-3} at the surface to 900 kg m^{-3} at 50 m and is then uniform to the bottom. Its temperature gradient is from 0°C at the surface to -30°C at 50 m and is then uniform to the bottom.

(a) Does the snow-ice transition in the iceberg (800 kg m^{-3}) occur above or below the waterline (assume sea water density is 1025 kg m^{-3})? What is the difference in terms of decay mechanisms?

(b) Calculate the total input of heat required to completely melt the iceberg.

(c) If 10,000 such icebergs per year are emitted from Antarctica and melt north of the Antarctic Convergence, estimate the poleward heat flux in the Southern Ocean due to icebergs, and compare with the total oceanic polar heat flux of 2 x 10^{15} W.

END OF PAPER