UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 29 May, 2009 9:00 am to 12:00 pm

PAPER 75

BIOLOGICAL PHYSICS

There are ***THREE*** *questions in total.*

ALL *questions should be answered.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Consider the reaction-diffusion system

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 1 + r\frac{u^2}{v} - u$$

$$\frac{\partial v}{\partial t} = p\frac{\partial^2 v}{\partial x^2} + q\left(u^2 - v\right)$$

where r, p, and q are positive constants. Find the fixed point of the spatially-homogeneous solutions and determine its stability. Determine the conditions for a Turing instability driven by the diffusive terms and the critical wavelength at onset.

2 Suppose a pattern u evolves according to

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - u(u-r)(u-1) \ ,$$

where $0 < r < 1$ is a parameter, with boundary conditions $u(-\infty) = 1$ and $u(\infty) = 0$. Find the stationary "front" for the case $r = 1/2$. When $|r - 1/2| = \epsilon \ll 1$ we expect the front to move slowly. Develop a perturbation theory for the front shape under the assumption $q_t/\sqrt{D} \sim \epsilon$, where $q(t)$ is the front location, and find the front velocity from a suitable solvability condition.

3 The end-to-end probability distribution function $\Phi(\mathbf{R}, N)$ of a polymer of N links is

$$\Phi(\mathbf{R}, N) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \cdots \int d\mathbf{r}_N \delta\left(\mathbf{R} - \sum_{n=1}^{N} \mathbf{r}_n\right) \Psi\left(\{\mathbf{r}_n\}\right) \ ,$$

where Ψ is the distribution of segment vectors $\mathbf{r}_n = \mathbf{R}_n - \mathbf{R}_{n-1}$. Assuming that Ψ is factorizable into a product of functions ψ for each segment, and using the freely-jointed-chain distribution $\psi(\mathbf{r}) = \delta(|\mathbf{r}| - b)/4\pi b^2$, where b is the step length, find Φ in the limit of large N.

The scattering function g is defined as

$$g(\mathbf{k}) = \frac{1}{N} \sum_{m,n}^{N} \langle \exp\left[i\mathbf{k} \cdot (\mathbf{R}_n - \mathbf{R}_m)\right] \rangle \ .$$

For a segment distribution function that depends only on the magnitude of the segment vector show that

$$g(\mathbf{k}) = \frac{1}{N} \sum_{m,n}^{N} \langle \frac{\sin\left(k|\mathbf{R}_n - \mathbf{R}_m|\right)}{k|\mathbf{R}_n - \mathbf{R}_m|} \rangle \ .$$

where $\langle \cdots \rangle$ denotes an average of the vector magnitude. Using the definition of the radius of gyration R_g

$$R_g^2 = \frac{1}{2N^2} \sum_{n=1}^{N} \langle (\mathbf{R}_m - \mathbf{R}_n)^2 \rangle \ ,$$

find the general small-k behaviour of g and the complete function for a Gaussian chain in scaling form

END OF PAPER