UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday, 3 June, 2009 1:30 pm to 4:30 pm

PAPER 65

GALAXIES

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

*This is an **OPEN BOOK** examination.*
Candidates may bring handwritten notes and lecture handouts into the examination.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 It can be shown that for a chemically evolving closed system the metal abundance Z by mass for newly formed stars can be expressed solely in terms of the metal yield y and the gas fraction $\mu = M_{\rm gas}/M$, as $Z = y \ln(1/\mu)$. Consider instead a system in which gas with metal abundance Z is ejected from the system at a rate that is a fraction ν of the star formation rate. For the sake of simplicity ignore the other gas recycling factor in the closed box model (i.e., set $R = 0$).

(i) Derive the analogous expression to the closed box case for the metal abundance Z as a function of μ, y, and ν.

(ii) Derive an expression for the number of stars as a function of metallicity $N(Z)$ in terms of y and ν.

(iii) Show that, if a galaxy begins forming stars from gas with zero metallicity, after the gas has been completely depleted and ejected the mean metal abundance of the stars is $< Z >= y/(1+\nu)$.

(iv) Spheroidal galaxies follow a mass-metallicity relation, with the most massive elliptical galaxies and bulges having approximately solar or supersolar metal abundances, and the lowest mass dwarf galaxies having abundances $Z < 0.01 Z_{\odot}$. What range in values of ν would be required to explain this range of stellar abundances?

(v) Briefly discuss the plausibility of the scenario in Part (iv) above in terms of the dynamical properties of galaxies. What scaling properties of galaxies would tend to produce variation in ν of the kind described above?

2 One of the key experiments on the recently launched Fermi (GLAST) gamma-ray observatory is a search for radiation from the possible decay of dark matter particles. Consider a galaxy of which the mass at large radii is dominated by a spherical dark matter halo.

(i) Derive an expression for the volume density $\rho(r)$ of the dark matter, as a function of radius r, for a galaxy with constant velocity disperson V.

(ii) Suppose that the dark matter particles decay spontaneously with some emissivity per particle ϵ. Derive an expression for the radial dependence of the projected surface brightness of the decay radiation. What is the radial dependence of the surface brightness at large radii?

(iii) Suppose that the dark matter particles emit radiation by annihilation, i.e., by two-body collisions between the particles. Derive the expression for the radial dependence of surface brightness of this radiation for this case. Again, what is the radial dependence of the surface brightness at large radii?

(iv) In the event that the Fermi data show a signal, it will be important to rule out a faint gamma-ray emission component associated with stars and/or gas in the disc, bulge, or halo of the galaxies. Explain (quantitatively) how one could use the radial dependence of the radiation to distinguish between emission from dark matter as opposed to stars or gas clouds?

3 A stellar system is composed of a population with an initial mass function (IMF) of the form $\psi(m)dm \propto m^{-3}dm$, and mass limits as given below.

(i) Consider first a galaxy with an old stellar population, with stars between masses of 0.1 and 1 $\mathrm{M}_\odot$. Derive the mass-to-light ratio of the system in solar units, if one assumes that the main-sequence luminosity of stars scales as the third power of their mass, and one ignores contributions to the mass and luminosity from other stellar types.

(ii) How does the derived M/L ratio compare to the values typically observed for old galaxies? To what extent are the assumptions above reasonable approximations and to what extent are they unrealistic?

(iii) Observations of the extended rotation curves of nearby disc galaxies show that the typical local M/L ratio in the outer halos of these galaxies must be of order 1000 $\left(\frac{M}{L}\right)_\odot$. In the past it was postulated that this dark matter could be composed of very low-mass substellar objects. If these objects followed the same inverse cubic IMF as given above, calculate what the lower mass limit of the IMF would have to be to produce a M/L ratio of 1000.

(iv) Briefly describe the observational constraints that rule out the presence of such massive substellar halos, at least for the Milky Way.

(v) Consider now a young gravitationally bound stellar system, with a power-law IMF slope $\psi(m) \propto m^{-\alpha}dm$, and mass limits of 0.1 to 100 $\mathrm{M}_\odot$. Suppose that all stars with a mass above 10 $M_\odot$ explode as supernovae, and (to simplify the problem) eject all of their mass from the stellar system. Estimate the limiting slope $\alpha_{\rm crit}$ of the IMF, beyond which the mass loss from supernovae will cause the system to become unbound.

[*HINT: This can easily be solved analytically, but you would need a calculator to solve for* $\alpha_{\rm crit}$. *Instead, derive the relative masses for integer values of the IMF slope* α *and interpolate to estimate the value of* $\alpha_{\rm crit}$.]

4 MINI ESSAY QUESTIONS. Explain in two or three sentences each the astrophysical explanations for each of these observations of galaxies and stellar populations. You may include equations if you wish but it is not required.

(i) Virtually every globular cluster in the Galaxy has nearly identical appearance and structure, whereas nearly every galaxy, including spheroidal galaxies, has a unique structure and appearance. Why?

(ii) A plot of numbers of binary galaxy systems as a function of mutual separation shows a marked depression for small separations less than about 100 kpc. Why is there a dearth of such small-separation pairs?

(iii) Stars in a spiral galaxy at a given radius orbit around the disc significantly more slowly than the gas. Why?

(iv) Most massive, flattened elliptical galaxies do not rotate. So why aren't they spherical?

(v) Although galaxies span a range of a factor of about 10^6 in luminosity, with faint dwarf galaxies being the most numerous by far, the vast majority of galaxies catalogued in large surveys such as the Sloan Digital Sky Survey have luminosities that lie within a range of less than a single decade (2.5 magnitudes). Explain why.

(vi) The radial luminosity profiles for most of the globular clusters and dwarf galaxies in the Galactic halo are sharply truncated at large radii. Why?

(vii) When one measures abundances of individual heavy elements for stars in the Galaxy, stars in the bulge and halo tend to be overabundant in certain elements (e.g., O, Mg, Ne) relative to iron, when compared to disc stars in the solar neighbourhood. What is thought to be the main explanation for this difference?

(viii) The most luminous infrared starburst galaxies show a limiting star formation rate of approximately $1000\,\mathrm{M_\odot\,yr^{-1}}$. Why?

END OF PAPER