UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 9 June, 2009 9:00 am to 11:00 am

PAPER 60

SUPERGRAVITY

*Attempt no more than **THREE** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

You may assume the following identities whenever needed:

$$\begin{aligned}\gamma^a\gamma^b\gamma^c &= \gamma^{[abc]} + \eta^{ab}\gamma^c - \eta^{ca}\gamma^b + \eta^{cb}\gamma^a \\ \gamma^{[abc]} &= \gamma^a\gamma^{[bc]} + \eta^{ac}\gamma^b - \eta^{ab}\gamma^c\,.\end{aligned}$$

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 A Rarita-Schwinger field ψ_μ in four dimensional Minkowski spacetime satisfies the equation of motion

$$\gamma^{\mu\nu\sigma}\partial_\nu\psi_\sigma = 0\,.$$

Show that the field carries two degrees of freedom and that these degrees of freedom are massless and have helicity $\frac{3}{2}$.

2 A Dirac spinor moving in a classical spacetime satisfies

$$\gamma^\mu\nabla_\mu\,\psi + m\,\psi \;=\; 0\,,$$

where ∇_μ is the torsion-free metric preserving covariant derivative. What equation is satisfied by $\gamma_5\psi$?

Show that ψ satisfies

$$-\nabla^2\,\psi + \frac{R}{4}\,\psi + m^2\,\psi \;=\; 0\,,$$

where R is the Ricci scalar.

How is this result modified if ∇_μ is replaced by a general metric preserving covariant derivative with torsion? What can you say about the coupling of spin to curvature?

3 Show, neglecting four fermion terms, that the action

$$\int |e| d^4\,x\big(\frac{R}{2\kappa^2} + \frac{1}{2}\bar\psi_\mu\gamma^{\mu\nu\sigma}D_\nu\psi_\sigma\big)$$

is invariant under

$$\delta\psi_\mu = \frac{1}{\kappa}\nabla_\mu\epsilon\,,\quad \delta e^a_\mu = \frac{\kappa}{2}\bar\psi_\mu\gamma^a\epsilon\,,$$

where

$$D_\nu\psi_\sigma = \partial_\nu\psi_\sigma + \frac{1}{4}\omega_{\nu ab}\gamma^a\gamma^b\psi_\sigma\,.$$

4 Explain what is meant by a Killing spinor field and its significance?

Show that if ϵ is a Killing spinor field of simple ungauged supergravity, then $K^\mu = \overline{\epsilon}\gamma^\mu\epsilon$ is an everywhere non-spacelike Killing vector field. Obtain all the solutions of the equations of motion if K^μ is timelike. Describe the solutions if K^μ is lightlike.

Show, assuming that it is of constant curvature, Anti-de-Sitter spacetime admits spinor fields satisfying

$$\nabla_\mu\epsilon + \frac{1}{2a}\gamma_\mu\epsilon = 0\,,$$

where a is a constant.

5 Write an essay on Witten's proof of the positive energy therem. Your essay should contain a derivation of the Witten identity from the Nester two-form and a justification of how the relevant boundary terms are related to the ADM mass.

END OF PAPER