UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 8 June, 2009 9:00 am to 12:00 pm

PAPER 25

SET THEORY AND LOGIC

Attempt no more than ***FOUR*** *questions.*

There are ***SEVEN*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Write an essay on computable functions $\mathbb{N}^k \rightharpoondown \mathbb{N}$. You should cover (among other things) Rice's theorem and the theorem of Jockusch that there is an infinite recursive partition of $[\mathbb{N}]^3$ with no infinite recursive monochromatic set.

2 Explain the device of Rieger-Bernays permutation models, and use it to prove the independence of the axiom of foundation from ZF. Extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

3 Prove Kruskal's theorem on wellquasiordering of trees and deduce Friedman's Finite Form of it.

4 Use Ramsey's theorem to prove (i) the Ehrenfeucht-Mostowski theorem, and (ii) the consistency of simple typed set theory with typical ambiguity and *urelemente*.

5 (a) What is a measurable cardinal? Explain the connection with elementary embeddings. Why is the canonical embedding into the transitive collapse of the ultrapower not the identity? What can you say about the first ordinal moved by it?

(b) State and prove the Erdös-Rado theorem on partitions with uncountable monochromatic sets. One consequence of this theorem is that a certain increasing function on ordinals is total; give a condition (the *tree property*) for a supremum of iterates of this function to be a fixed point for it. Prove that any cardinal with the tree property must be strongly inaccessible.

6 Give a proof of the Ehrenfeucht-Mostowski theorem using ultraproducts not Ramsey's theorem.

7 Prove the lemma of Frayne's that elementarily equivalent structures have isomorphic ultralimits.

END OF PAPER