UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 3 June 2008 1.30 to 3.30

PAPER 77

COMPUTER AIDED GEOMETRIC DESIGN

Attempt no more than ***FOUR*** *questions.*

There are ***SIX*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (i) Describe an algorithm to determine whether two triangles in 3-space intersect. Each triangle is defined by the three coordinates of each of its three vertices.

(ii) How many arithmetic operations (+, -, *, /) are required ? (For this purpose you may assume that the triangles are small compared with the size of a box that they are randomly distributed in.)

(iii) If intersection of two objects, each consisting of many ($O(10^5)$) triangles is required, how can the process be made more economical than just repeating the simple check between two triangles ?

2 Compute, in approximating polygonal form (within a given ϵ), the curve on a parametric surface $P(u, v)$, $u, v \in [0, 1]$, at which the reflection of a parametric curve $Q(t)$, $t \in [0, 1]$ is seen by an observer at point E.

You may assume that the normal at every point of P has a component towards E.

3 (i) Identify different ways of evaluating a point at a particular parameter value on a B-spline curve.

(ii) What are the relevant advantages of one method over another ?

(iii) A uniform cubic B-spline control polygon contains the sequence of control points

	x	y	z
$\vdots$			
P_1	1	1	1
P_2	1	0	1
P_3	0	0	0
P_4	0	1	0
P_5	-1	1	1
P_6	-1	0	1
$\vdots$			

Evaluate the point and first derivative at $t = 2\frac{3}{4}$.

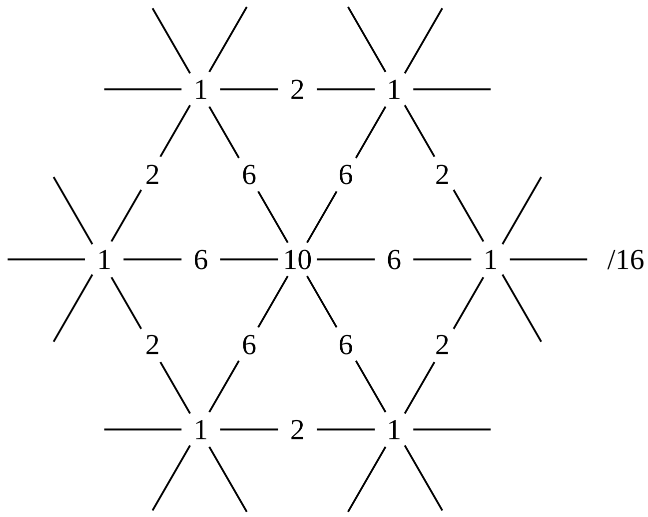
4 (i) Why is it useful to be able to offset a surface ?

(ii) How can a surface defined as being offset from a parametric surface be interrogated as a parametric surface ?

(iii) How can a surface defined as being offset from a subdivision surface be interrogated as a subdivision surface ?

(iv) What is a practical (useful but also practicable) class of offset forms ?

5 "Loop" is a uniform, stationary, binary bivariate subdivision scheme defined over a triangulation by the mask



(i) Identify the stencils for the vertices after one refinement step.

(ii) Prove that if the scheme is applied to functional data ($z = f(x, y)$) which is extruded along one of the grid edge directions, the limit surface is extruded.

(iii) Identify the mask of the appropriate univariate scheme to be applied along edges.

(iv) What is the maximum degree d of polynomial such that if the original vertices lie on that polynomial the limit surface will be the same polynomial ?

(v) What is the maximum degree d of polynomial such that if the original vertices lie on that polynomial, the limit surface will be a polynomial of the same degree ?

6 Compute, in approximating polygonal form (within a given ϵ), the curve on a subdivision surface S, at which the reflection of a given subdivision curve C is seen by an observer at point E.

END OF PAPER