UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 30 May 2008 9.00 to 12.00

PAPER 59

PHILOSOPHY OF CLASSICAL AND QUANTUM MECHANICS

*Attempt **THREE** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

Substantial repetition of material in different questions will be penalized.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Write an essay about how a theory of some class of emergent or collective phenomena can emerge in the limit of some theory as a parameter goes to some special value, e.g. the number of degrees of freedom goes to ∞. To obtain full credit the essay should:

(i) discuss the relations of reduction and emergence (between theories, or phenomena, or both); and

(ii) illustrate the issues using either (a) fractals, or (b) phase transitions (treated either with classical or with quantum theory), or (c) superselection in quantum statistical mechanics.

2 Write an essay about a set of assumptions for a two-particle Bell experiment that are sufficient for a deduction of a Bell inequality. To obtain full credit the essay should:

(i) consider formulating the assumptions in terms of the state of, or physical information within, spacetime regions (using either (a) the framework of 'local causality' in Bell's sense, or (b) the framework of 'common causes' or (c) the framework of 'stochastic Einstein locality');

(ii) state which assumptions you consider to be false, and give reasons for your verdict.

3 Write an essay about the formulations, in modern geometric language, of the Lagrangian and Hamiltonian mechanics of a finite dimensional system. To obtain full credit the essay should:

(i) state and discuss, for both the Lagrangian and Hamiltonian formulations, Noether's theorem relating symmetries and conserved quantities.

(ii) discuss the significance of the translational and rotational symmetry of Newtonian gravitational theory, and the quotienting (or elimination of variables) that it licenses, for the debate about absolute vs. relational conceptions of space.

4 Write an essay about the modern theory of Poisson manifolds. To obtain full credit the essay should include a discussion of the following topics: the actions of Lie groups on manifolds, and the quotient structures that thus arise; the adjoint representation of a Lie group G on its Lie algebra g; the co-adjoint representation of G on the dual g^*; the Poisson structure of g^*; the symplectic foliation of Poisson manifolds; the example of $so(3)$ and $so(3)^*$ and their use to describe the rigid body.

5 Write an essay discussing the various formulations of locality for a quantum theory. To obtain full credit the essay should include a discussion of at least three of the following five topics (the last three of which apply to a quantum field theory):

(i) the spreading of wave-packets;

(ii) the quantum no-signalling theorem;

(iii) spacelike commutation of observables;

(iv) the spectrum condition, that the spectrum of the energy-momentum operator lies in the future light-cone;

(v) primitive causality, that the state's restriction to a spacetime region O determines the expectation values of all operators associated with points or subregions of the future domain of dependence, $D^+(O)$, of O.

END OF PAPER