UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 10 June 2008 1.30 to 3.30

PAPER 56

SUPERGRAVITY

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Outline the Noether principle for obtaining an action invariant under a local symmetry from one invariant under a global symmetry. Sketch how it can be used to construct the on-shell supergravity Lagrangian, where

$$\mathcal{L} = -\frac{1}{2\kappa^2}\, e\, R - \frac{1}{2}\, \epsilon^{\mu\nu\rho\sigma}\, \overline{\Psi}_\mu\, \gamma_5\, \gamma_\nu\, \tilde{D}_\rho\, \Psi_\sigma\,,$$

where e is the determinant of the vierbein and $\tilde{D}_\rho$ the covariant derivative. How does the covariant derivative differ from the usual one in general relativity?

2 Show that the Rarita–Schwinger equation,

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma = 0$$

can be used to describe the gravitino. What is the spin of Ψ_μ?

Compute the number of off-shell degrees of freedom for a massless gravitino in four dimensions. Compare this to the corresponding number of degrees of freedom for the graviton. Why are auxiliary fields introduced in supergravity?

3 Consider the Polonyi superpotential

$$W = \mu\, m_p\, (z + \beta)\,,$$

where z is a complex field, β a real constant, μ a mass parameter and m_p is the Planck mass. Taking the Kähler potential

$$K = z^* z\,,$$

compute the scalar potential, V.

For $\kappa\beta = 2 - \sqrt{3}$ and $\kappa z = \sqrt{3} - 1$, where $\kappa = m_p^{-1}$, show that your potential has a minimum at $V = 0$. Show that supersymmetry is broken. Why is the gravitino mass small compared to the supersymmetry breaking scale?

4 An $N = 1$ supergravity theory has three chiral superfields with superpotential

$$W = \lambda \Phi_0 \Phi_+ \Phi_- \,,$$

Kähler potential

$$K = |\Phi_0|^2 + |\Phi_+|^2 + |\Phi_-|^2 \,,$$

and D-term

$$V_D = g \left(\phi_+^* \phi_+ - \phi_-^* \phi_- - \zeta \right)^2 \,.$$

Compute the full potential and show that

$$V = V_F + V_D \geqslant 0 \,.$$

Show that there are two minima, giving the vacuum expectation value of the fields in the supersymmetric vacuum. Briefly discuss any differences between D-term supersymmetry breaking in supergravity and the global case.

END OF PAPER