MATHEMATICAL TRIPOS Part III

Wednesday 4 June 2008 1.30 to 4.30

PAPER 53

SUPERSYMMETRY AND EXTRA DIMENSIONS

Attempt **ALL** *questions.*

There are **FOUR** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Write an essay about the hierarchy problem of the standard model of particle physics, including a discussion of how supersymmetry addresses the different aspects of the problem and comparing it with extra dimensions as alternative solutions.

2 State and prove the non-renormalisation theorem for the superpotential W and the gauge kinetic function f of $N = 1$ supersymmetric theories.

3 Consider a general antisymmetric tensor field of rank p in D dimensions.

(a) Establish a duality with an antisymmetric tensor of rank q for both the massless and massive cases. Determine the value of q in each case.

(b) Count the number of degrees of freedom for a massless antisymmetric tensor of rank p in $D > 4$ dimensions. Dimensionally reduce it to four dimensions and verify the matching of the total number of degrees of freedom.

(c) Dimensionally reduce the *bosonic* massles field content of 11-dimensional supergravity in terms of the fields: G_{MN}, A_{MNP}, $(M, N, P = 0, 1, \cdots, 10)$, to ten dimensions and to four dimensions and verify the matching of the number of degrees of freedom.

4 Consider the $N = 1$ supersymmetry algebra.

(a) Derive the part of the algebra that includes the generators Q_α and/or $\bar{Q}_{\dot{\beta}}$.

(b) Use this algebra to show that, within a supersymmetry multiplet, the number of bosons equals the number of fermions and that supersymmetry is broken if and only if the energy of the vacuum is positive definite.

(c) Find the helicity structure of the massless multiplets.

(d) Which new ingredients to the algebra are introduced in the extended supersymmetry case $(N > 1)$? Assuming there are no massless particles of helicity $\lambda > 2$ show that the maximum number of supersymmetries is $N = 8$.

END OF PAPER