MATHEMATICAL TRIPOS Part III

Monday 9 June 2008 1.30 to 4.30

PAPER 27

SET THEORY AND LOGIC

Attempt no more than **FOUR** *questions.*

There are **NINE** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Give two proofs of the Ehrenfeucht-Mostowski theorem, one using Ramsey's theorem and the other using ultraproducts.

2 Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

3 Write an essay on the theory of computable functions, setting out all the background necessary to prove Rice's theorem (which you should prove).

4 (a) State and prove the completeness theorem for first order logic.

(b) State and prove Łoś's theorem. Use it to give an ultraproduct proof that if T is a theory all of whose finite fragments have models then T has a model.

5 (a) State and prove Gödel's theorem on the incompleteness of arithmetic.

(b) Explain Shepherdson's wall, and prove the consistency of the Axiom of Choice relative to ZF.

6 Prove the consistency of NFU.

7 What is a WQO? A BQO? State and prove Kruskal's theorem on wellquasiorderings of trees. Deduce Friedman's Finite Form from it.

8 Use the method of inner models to show that classical ZF is no stronger than constructive ZF.

9 Prove Frayne's theorem that two first-order structures are elementarily equivalent iff they have isomorphic ultralimits.

END OF PAPER