UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 3 June 2008 1.30 to 4.30

PAPER 23

ALGEBRAIC GROUPS

All questions are of equal weight.

Answer ***either*** *Question 1* ***or*** *Question 2*

and *any 2 others.*

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Define the *Lie algebra* of a Lie group and of a complex affine algebraic group. Compute the Lie algebras of U_n, $GL_n(\mathbb{C})$, SU_n and $SL_n(\mathbb{C})$. Explain what it means for an irreducible complex affine algebraic group to be reductive, and give some examples, with justification.

2

Suppose that G is either a compact Lie group or a reductive complex algebraic group and that V is a finite dimensional complex representation of G.

(i) Show that V is completely reducible.

(ii) Show that, if $\mathbb{C}[V]$ is the ring of polynomial functions on V, then the ring of invariants $\mathbb{C}[V]^G$ is finitely generated.

3

Describe the symbolic method for the group $SL_2(\mathbb{C})$, and describe the representations of $SL_2(\mathbb{C})$.

4

Define the notion of an *unstable* point in a representation V of $SL_n(\mathbb{C})$. State and prove a characterization of unstable points in terms of one-parameter subgroups of G, and describe the unstable points in the representation $\mathrm{Symm}^6(\mathrm{W})$ of $SL_2(\mathbb{C})$, where W is the standard 2-dimensional representation.

5 Suppose that k is an algebraically closed field of characteristic $p > 0$. Prove that tori are linearly reductive and that $SL_2(k)$ is geometrically reductive. (Define your terms.)

END OF PAPER