UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 4 June 2007 1.30 to 4.30

PAPER 63

BLACK HOLES

Attempt ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Starting from the line element $ds^2 = g_{ab}dx^a dx^b$, explain how to find an orthonormal basis of 1-froms $E^{\hat{a}}$. How does one find the connection 1-form, curvature 2-form, Riemann tensor and Ricci tensor from $E^{\hat{a}}$.

A spherically symmetric static spacetime has line element

$$ds^2 = -V(r)\,dt^2 + \frac{dr^2}{V(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Calculate the curvature 2-form and hence find the components of the Ricci tensor.

The Einstein equation for a spacetime with a cosmological constant $\Lambda = -3/l^2$ is

$$R_{ab} = -\frac{3}{l^2}g_{ab}.$$

Find $V(r)$. Explain why the metric you find either has or does not have an event horizon.

2 In string theory, the analogue of the Reissner-Nordstrom black hole of mass M and charge Q has line element

$$ds^2 = -\frac{(1-2M/r)}{(1-Q^2/Mr)}\,dt^2 + \frac{dr^2}{(1-2M/r)(1-Q^2/Mr)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where you may assume that $Q^2/2M^2 < 1$.

Where is the event horizon?

Use Euclidean techniques to determine the temperature of the black hole, carefully justifying your argument by using the fact that e^{iHt} is the quantum mechanical time translation operator for a system with Hamiltonian H.

Suppose one has an electrically neutral solar mass black hole, ($1M_\odot = 2.10^{33}$gm). Estimate how long, in years, it takes before the black hole can be expected to evaporate completely.

3 A beacon pulses at a time interval M emitting a monochromatic pulse of duration τ and wavelength λ where $\lambda \ll \tau \ll M$. It starts to fall radially into a Schwarzschild black hole of mass M starting from rest relative to the hole from $r = 8M$. (r is the usual Schwarzschild radial coordinate).

How long does it take the beacon to reach the horizon?

An observer at rest relative to the hole and at a very large distance from it watches the beacon fall in. How many pulses does the observer see?

What is the approximate observed duration of the last pulse?

What is the approximate observed frequency of the last pulse?

4 Find the absorbtion cross-section for light by a Reissner-Nordstrom black hole of mass M and charge Q.

Check that your answer agrees with the classical result for the Schwarzschild black hole.

[*Hint: You may find it helpful to express your result in terms of M and ψ where*

$$Q = \frac{3M \sin\psi}{\sqrt{8}}.$$

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END OF PAPER