UNIVERSITY OF
CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 12 June 2007 9.00 to 11.00

PAPER 57

SUPERGRAVITY

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Outline briefly the Noether principle for obtaining an action invariant under a local symmetry from one invariant under a global symmetry. Sketch how it can be used to couple the Wess-Zumino Lagrangian

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* + \frac{i}{2}\,\overline{\psi}\gamma^\mu\partial_\mu\psi\,,$$

where $\phi = \frac{1}{\sqrt{2}}(A+iB)$ is a complex scalar field and ψ a Majorana fermion, to supergravity. The final result is

$$\begin{aligned}\mathcal{L} = {} & e\partial_\mu\phi\partial^\mu\phi^* + \frac{i}{2}\,e\overline{\psi}\gamma^\mu D_\mu\psi \\ & - \frac{1}{2\kappa^2}\,eR - \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\overline{\Psi}_\mu\gamma_5\gamma_\nu D_\rho\Psi_\sigma \\ & - \frac{\kappa}{2}\,e\overline{\Psi}_\mu\phi(A - i\gamma_5 B)\gamma^\mu\psi \\ & - \frac{i}{4}\kappa^2\,\epsilon^{\mu\nu\rho\sigma}\overline{\Psi}_\mu\gamma_\nu\Psi_\rho A\overleftrightarrow{D}_\sigma B \\ & - \frac{\kappa^2}{4}\,e\overline{\psi}\gamma_5\gamma^\mu\psi A\overleftrightarrow{D}_\mu B\,,\end{aligned}$$

where e is the determinant of the vierbein. Discuss the origin of each term, no mathematical derivations are necessary.

2 The Rarita–Schwinger equation for a massless field is

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma = 0\,.$$

Show that this is the equation of motion for a spin $\frac{3}{2}$ field. How many degrees of freedom does Ψ_μ have? If a mass term were introduced into the above equation, would the degrees of freedom change?

By comparing the number of degrees of freedom for Ψ_μ with those for the graviton explain why auxiliary fields are introduced in supersymmetry.

3 In $N=1$ supergravity three chiral superfields interact with superpotential

$$W = g\Phi_0(\Phi_+\Phi_- - \zeta)\,,$$

where ζ is a constant.

The Kähler potential is

$$K = |\Phi_0|^2 + |\Phi_+|^2 + |\Phi_-|^2\,.$$

In the limit $|\Phi_\pm|^2 \ll M_p^2$, where M_p is the reduced Planck mass, calculate the effective potential. Show that the potential has two minima, one where supersymmetry is broken and a supersymmetric minima. How does the potential differ from that obtained in the corresponding global case?

4 An $N=1$ supergravity theory has two chiral superfields with Kähler potential (in Planck units)

$$K = -3\log(T+T^*) + C^*C\,,$$

and superpotential

$$W = C^3 + B\,,$$

where B is a complex constant. Calculate the effective potential. By considering the auxiliary field, verify that supersymmetry is broken. Are there any flat directions? What is the vacuum energy at the minimum? Calculate the gravitino mass.

END OF PAPER