UNIVERSITY OF CAMBRIDGE

# MATHEMATICAL TRIPOS Part III

Thursday 7 June 2007 1.30 to 4.30

## PAPER 55

## SUPERSYMMETRY AND EXTRA DIMENSIONS

*Attempt **QUESTION 1 AND ANY OTHER THREE** questions.*

*There are **FIVE** questions in total.*

*The questions carry equal weight.*

***STATIONERY REQUIREMENTS***
*Cover sheet*
*Treasury Tag*
*Script paper*

***SPECIAL REQUIREMENTS***
*None*

**You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.**

**1** State clearly the hierarchy and cosmological constant problems. Argue how supersymmetry could be used to address each of these problems separately and explain why it is usually only used for the first one. Describe in detail two higher dimensional proposals to address the hierarchy problem and mention the main challenge they have to actually solve this problem.

**2** Find the coefficient of $\theta\theta$ of the combination $W = \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3$ where $\Phi$ is a chiral superfield and $m, g$ are constants.

Find the minima of the scalar potential $V = |\partial W/\partial\Phi|^2$ and, expanding around one of the minima, show explicitly that the mass of the scalar field equals the mass of the fermion field and that the quartic self-coupling of the scalar field equals the Yukawa coupling among this field and the fermion.

Use symmetry arguments and holomorphy to show that $W$ does not receive perturbative quantum corrections.

**3** Consider a renormalisable $N = 1$ supersymmetric theory with chiral superfields $\Phi_i = (\varphi_i, \psi_i, F_i)$ and vector superfields $V_a = (\lambda_a, A_a^\mu, D_a)$ with both D and F term supersymmetry breaking ($F_i \neq 0,\ D_a \neq 0$). Show that

$$\frac{\partial V}{\partial \varphi^i} = F^j \frac{\partial^2 W}{\partial \varphi^i \varphi^j} - g^a D^a \varphi_j^\dagger \left(T^a\right)_i^j = 0,$$

in the vacuum. Here $g^a$ and $T^a$ refer to the gauge coupling and generators of gauge group respectively and $D^a = -g^a \left(\varphi_i^\dagger \left(T^a\right)_j^i \varphi^j + \xi^a\right)$ with $\xi^a$ the Fayet-Iliopoulos term. Since the superpotential $W$ is gauge invariant, the gauge variation of $W$ is

$$\delta^{(a)}_{gauge} W = \frac{\partial W}{\partial \varphi^i} \delta^{(a)}_{gauge} \varphi^i = F_i^\dagger \left(T^a\right)_j^i \varphi^j = 0.$$

Write these two conditions in the form of a matrix $M$ acting on a 'two-vector' with components $\langle F^j \rangle$ and $\langle D^a \rangle$. Identify this matrix and show that it is the same as the fermion mass matrix. Argue that it has one zero eigenvalue which can be identified with the Goldstone fermion.

Consider a renormalisable $N = 1$ supersymmetric Lagrangian for chiral superfields with F-term supersymmetry breaking. By analysing the mass matrix for scalars and fermions show that

$$\mathrm{STr} M^2 \equiv \sum_j (-1)^{2j+1} \left(2j+1\right) m_j^2 \ = \ 0$$

where $j$ represents the 'spin' of the particles. What implication this result could have for the MSSM?

**4** Starting from the field content of 10D super Yang-Mills: an $N = 1$ gauge field and its gaugino partner, dimensionally reduce it to 4D describing the number of supersymmetries and the structure of the multiplet. What form does the scalar potential take in the 4D effective action? Count the number of degrees of freedom for an antisymmetric tensor field $A_{MNP}$ in $D \geqslant 6$ dimensions, dimensionally reduce it to 4D and verify the matching of the number of degrees of freedom. Dimensionally reduce the field content of 11D supergravity to 10D and 4D.

**5** Let us consider the field content of the MSSM, transforming under $SU(3)\times SU(2)\times U(1)_Y$ as:

$$\begin{aligned} Q_i &= (3,2,-\tfrac{1}{6}), & \bar{u}^c_i &= (\bar{3},1,\tfrac{2}{3}), & \bar{d}^c_i &= (\bar{3},1,-\tfrac{1}{3}), \\ L_i &= (1,2,\tfrac{1}{2}), & \bar{e}^c_i &= (1,1,-1), & \bar{\nu}^c_i &= (1,1,0), \\ H_1 &= (1,2,\tfrac{1}{2}), & H_2 &= (1,2,-\tfrac{1}{2}), \end{aligned}$$

where the indices $i = 1,2,3$ label the three different generations.

Write down the most general cubic superpotential for these fields, invariant under the symmetries of the standard model. Separate the terms that preserve baryon and lepton number from those that do not preserve it. Show that combining two of the baryon/lepton number violating terms will induce proton decay: $p \to e^+ + \pi^0$. Estimate the decay rate of the proton via this channel based on dimensional grounds.

The experimental lower bound on the proton lifetime is approximately:

$$\tau_{\text{proton}} > 5.5 \times 10^{32} yrs = 1.6 \times 10^{40} s = 2.4 \times 10^{64} GeV^{-1}$$

Use this to determine an upper bound on the product of the two 'Yukawa' couplings that give rise to proton decay above.

Define $R$-parity and show how it acts on each of the particles of the standard model. Verify that $R$-parity precisely forbids all baryon/lepton number violating terms while preserving the fermion mass terms. State clearly two important physical implications of R-parity.

## END OF PAPER