UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 11 June, 2007 9 to 12

PAPER 53

THE STANDARD MODEL

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Consider an effective Hamiltonian matrix for kaons

$$\begin{pmatrix} \langle K^0|H'|K^0\rangle & \langle K^0|H'|\bar{K}^0\rangle \\ \langle \bar{K}^0|H'|K^0\rangle & \langle \bar{K}^0|H'|\bar{K}^0\rangle \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}.$$

Draw one-loop Feynman diagrams for $\langle \bar{K}^0|H'|K^0\rangle$ and estimate a rough order of magnitude for it in terms of fundamental Standard Model parameters. Derive a relationship between the matrix elements assuming CPT invariance. What relationship between the matrix elements would CP invariance imply? How are the CP eigenstates $|K_1^0\rangle$ (CP=+1) and $|K_2^0\rangle$ (CP=−1) defined in terms of $|K^0\rangle$, $|\bar{K}\rangle$?

Assuming CPT, obtain the mass eigenstates $K^0_{S,L}$ at the time of their production in terms of the CP-eigenstates $K^0_{1,2}$

$$|K_S{}^0\rangle = \frac{1}{(1+|\epsilon|^2)^{\frac{1}{2}}}\Big(|K_1{}^0\rangle + \epsilon|K_2{}^0\rangle\Big),$$

$$|K_L{}^0\rangle = \frac{1}{(1+|\epsilon|^2)^{\frac{1}{2}}}\Big(|K_2{}^0\rangle + \epsilon|K_1{}^0\rangle\Big),$$

where

$$\epsilon = \frac{\sqrt{M_{12}} - \sqrt{M_{21}}}{\sqrt{M_{12}} + \sqrt{M_{21}}}.$$

2 Neglecting fermion masses, calculate the spin and colour-averaged matrix element squared $|\bar{\mathcal{M}}|^2$ for $e^+(p_1)e^-(p_2) \to q(q_1)\bar{q}(q_2)$ at tree-level in QED in terms of the charge of the quark Q_q, the fine structure constant α and $s = (p_1 + p_2)^2$:

$$|\bar{\mathcal{M}}|^2 \propto (p_1.q_2 \; p_2.q_1 + p_2.q_2 \; p_1.q_1).$$

Find the constant of proportionality. By considering the kinematics in the centre of mass frame, derive the total cross section

$$\sigma = \frac{4\pi\alpha^2}{s} Q_q^2.$$

Describe briefly how this relates to the cross-section for $e^+(p_1)e^-(p_2) \to$ hadrons.

[*You may use the Feynman gauge propagator for a photon with 4-momentum p^μ, i.e. $-ig_{\mu\nu}/(p^2 + i\epsilon)$. You may also use*

$$\tfrac{d\sigma}{d\Omega} = \frac{|\bar{\mathcal{M}}|^2}{64\pi^2 p_1.p_2}$$

in the centre of mass frame, where Ω is the solid angle subtended between e^+ and $\bar{q}$. You may find the following trace identity useful

$$tr(\gamma^\mu\gamma^\sigma\gamma^\rho\gamma^\nu) = 4(g^{\mu\sigma}g^{\rho\nu} - g^{\mu\rho}g^{\sigma\nu} + g^{\mu\nu}g^{\sigma\rho})$$]

3 Write an essay on quark mixing in the Standard Model.

4 A three-component triplet gauge field $\mathbf{A}_\mu$ is coupled to a real triplet scalar field $\mathbf{\Phi}$ with the Lagrangian density,

$$\mathcal{L} = -\tfrac{1}{4}\mathbf{F}^{\mu\nu}\cdot\mathbf{F}_{\mu\nu} + \tfrac{1}{2}(D^\mu\mathbf{\Phi})\cdot D_\mu\mathbf{\Phi} - \frac{1}{8}\lambda\big(\mathbf{\Phi}^2 - v^2\big)^2,$$
$$\mathbf{F}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu - e\,\mathbf{A}_\mu \times \mathbf{A}_\nu,$$
$$D_\mu\mathbf{\Phi} = \partial_\mu\mathbf{\Phi} + e\,\mathbf{A}_\mu \times \mathbf{\Phi}\,.$$

What gauge symmetry does this Lagrangian possess? What gauge symmetry does the ground state possess? Rewrite the theory in terms of physical fields and determine their masses. Which value of v leaves the original symmetry unbroken? Briefly discuss the number of degrees of freedom in the theory with and without spontaneous symmetry breaking. Draw and write the Feynman rules for the various interaction vertices involving physical scalar fields.

END OF PAPER