UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 8 June 2007 1.30 to 4.30

PAPER 23

ALGEBRAIC CURVES

*Attempt **THREE** questions.*

*There are **SIX** questions in total.*

The questions carry equal weight.

In answering any question, you may assume any result that you would have proved in answering a previous question.
Throughout, k is a field.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Describe $\mathbb{P}^n_{\mathbb{Z}}$ in terms of a functor, via quotients of a free module of rank $n+1$.

Use your answer to derive the Euler sequence for the tangent sheaf of $\mathbb{P}^n$ and compute $\Omega^1_{\mathbb{P}^1}$.

2 Describe the computation of $H^i(\mathbb{P}^1_k, \mathcal{O}(n))$ in terms of a Čech complex, and deduce a duality theorem for the cohomology of locally free coherent sheaves on $\mathbb{P}^1_k$.

3 Prove that any locally free coherent sheaf $\mathcal{E}$ on $\mathbb{P}^1_k$ is of the form $\mathcal{E} \cong \mathcal{O}(a_1) \oplus \ldots \oplus \mathcal{O}(a_n)$.

4 Show that any smooth projective curve C over k has a dualizing sheaf D_C (for locally free sheaves) and that $D_C \cong \Omega^1_C$. [*Any algebraic identities you need may be quoted without proof.*]

5 Suppose that C is a smooth projective curve. Show that $H^i(C, \mathcal{E}) = 0 \; \forall i \geqslant 2$ and for all locally free sheaves $\mathcal{E}$ on C.

Define the genus of C, and state and prove the Riemann–Roch theorem for curves.

6 Assume $k = \overline{k}$. Define the functors $\underline{Pic}^d_C$. Show how to represent them. Any results you need about the semi-continuity of cohomology groups may be assumed.

END OF PAPER