UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 4 June 2007 1.30 to 4.30

PAPER 20

SPECTRAL GEOMETRY

*Attempt **THREE** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Give three alternative definitions of the Laplacian acting on the functions on a Riemannian manifold and prove their equivalence.

2 Prove the existence of a 3-parameter family of pairs of bounded planar domains that are not isometric but are isospectral for the Laplacian acting on functions with Dirichlet boundary condition.

3 Define the heat kernel for a compact Riemannian manifold. If $p : N \to M$ is a finite normal locally isometric covering of Riemannian manifolds with covering transformation group U, obtain an expression for the heat kernel of M in terms of that of N.

Deduce a formula for the heat trace of M when U is a subgroup of a larger group T of isometries of N and hence deduce Sunada's Theorem.

4 Identify, with proof, a pair of Gassman equivalent subgroups U_1, U_2 of $PSL(3,2)$. Explain briefly why they are not conjugate.

Assuming any result you require about hyperbolic tri-rectangles, show how to construct an uncountable family of pairs of isospectral Riemann surfaces of genus four. Explain briefly why they are isospectral and not isometric.

[You may use the fact that $PSL(3,2)$ has generators A and D with commutator $C = [D,A]$ of order 7 such that the permutation actions of A and D on the cosets $U_i C^n$ are given by

Generator	cosets of U_1	cosets of U_2
A	(0)(1 2 5)(3 6 4)	(0)(1 4 3)(2 5 6)
D	(1)(0 3)(2 6 4 5)	(4)(2 5)(0 1 6 3)

where, in each case, the coset $U_i C^n$ is denoted by n.]

5 Define the Riemann surfaces referred to as X-pieces and Y-pieces. Identify the parameters that determine them up to isometry, including those involved when two Y-pieces form an X-piece.

Describe how a general closed Riemann surface of genus g may be formed from a certain number of Y-pieces and identify a set of parameters that suffice to determine the surface up to isometry.

Define Teichmüller space and state Wolpert's Theorem. Give three major ingredients of the proof of the theorem with a brief indication of the role that they play.

END OF PAPER