UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 8 June, 2006 1.30 to 4.30

PAPER 83

GEOLOGICAL FLUID MECHANICS

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Consider a porous medium bounded by two horizontal, impermeable planes separated by a distance h which is very much less than the horizontal extent of the medium. The top plane is held at a temperature T_0 and both planes are perfectly conducting. Making the Boussinesq approximation and assuming a linear relationship between density and temperature, obtain an expression for the maximum temperature of the lower plane, T_m, for which all small disturbances decay.

The space within a closed, vertical cylinder whose cross-section is a square is filled with a porous medium. The height of the cylinder is very much larger than the horizontal dimensions. Considering the vertical walls to be insulated and making the same assumptions as before, calculate the maximum decrease in vertical temperature per unit increase in height that is stable to small disturbances. Describe the form of motion that you would expect just beyond this critical temperature gradient.

2 Consider the instantaneous release of dense liquid from a horizontal line source at the base of an infinitely deep sedimentary rock layer lying above a thin, much less permeable shale band. A volume V per unit length of the liquid, which has density ρ, is released at time $t = 0$ into the rock layer which is to be modelled as a homogeneous porous medium saturated with liquid of density $\rho - \Delta\rho$ (with $\Delta\rho > 0$). The underlying shale band permits the released liquid to drain from the rock layer with a vertical velocity $v = \gamma h$, where γ is a constant, $h(x, t)$ is the thickness of the resulting liquid current in the rock and x is the horizontal co-ordinate.

Stating carefully any assumptions you make, show that the propagation of the dense liquid is governed by

$$h_t = \beta(hh_x)_x - \lambda h \ , \qquad (*)$$

where the constants β and λ are to be related to the physical parameters.

Consider first the case where the shale band is totally impermeable ($\lambda \equiv 0$). Writing down any appropriate boundary conditions, determine the appropriate similarity solution to (*). Evaluate explicitly the length of the current $L(t)$ as a function of time.

Consider now $\lambda \neq 0$. Write down an expression for the temporal rate of change of the total volume of released liquid in the permeable rock and hence determine an expression for this volume as a function of time. Introducing new variables

$$H = he^{\lambda t} \quad \text{and} \quad \tau = (1 - e^{-\lambda t})/\lambda$$

or otherwise, determine the solution of (*). Evaluate the length of the current as a function of time and interpret your result.

3 A short lived but violent volcanic eruption from a circular vent in flat countryside results in a hot and heavy ash-laden particulate flow. Write down the appropriate shallow water equations and boundary conditions governing the propagation of the flow assuming that all the ash particles are of one size, and neglecting any effects due to entrainment of the (cold) atmosphere of the flow. State clearly any assumptions you make.

Show how to reduce the complexity of these equations by assuming that the flow is horizontally uniform (i.e. by making a box model assumption). State clearly any further assumptions you have made.

Neglecting any effects due to the difference in density between the hot *air* in the flow and the atmosphere, determine the furthest point attained by ash. Evaluate also the time taken to reach this point and the resultant density of deposit of the ash.

Comment briefly on how you think the results would change if the ash in the flow was considered to be made up of:

(i) two widely spaced size distributions;

or

(ii) a continuous size distribution of ash particles.

4 A semi-infinite vertical plate is placed in an infinite porous medium and a constant heat flux F per unit horizontal width is applied to the plate. Evaluate the form of the boundary layer which develops and determine a single ordinary differential equation and the accompanying boundary conditions which describe the flow.

Obtain an approximate solution to this differential system.

Write down expressions for the total mass, momentum and buoyancy fluxes in the boundary layer. Evaluate the total momentum and buoyancy fluxes exactly, and the total mass flux using the approximate solution you have determined.

END OF PAPER