UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 6 June, 2006 9 to 11

PAPER 75

SOLIDIFICATION OF FLUIDS

ALL *questions may be attempted,*
full marks can be obtained by substantially complete answers to ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

Candidates may bring into the examination any lecture notes made during the course and any materials that were distributed during the course.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 A long vertical mould initially contains a layer of depth h of salt solution of concentration C_0 sandwiched between pure ice below and pure water above. It is pulled vertically downwards at speed V through a fixed temperature gradient G. The solution has liquidus $T = -mC$, where T is temperature, C is salt concentration and m is a constant, and eutectic temperature and concentration T_E and C_E respectively.

What is the resulting steady salt distribution if $V < DC_E/hC_0$ and the solid–liquid interface remains planar, where D is the diffusivity of salt in solution? What is the temperature of the interface? Describe qualitatively what would happen in an experiment if $V > DC_E/hC_0$.

Determine the growth rates of morphological instabilities of the solid–liquid interface, employing the frozen-temperature approximation, including the Gibbs-Thompson effect and assuming instantaneous kinetics.

Describe qualitatively the sequence of events following instability.

2 A mushy layer grows steadily vertically upwards at speed V. The solid–mush interface, at $z = 0$, is maintained at the eutectic temperature T_E. A straining flow in the liquid above the mushy layer imposes a pressure $p_0 - \frac{1}{2}E^2x^2$ and a uniform vertical temperature gradient G at the horizontal mush–liquid interface, $z = h$, where x is the horizontal coordinate. The liquid entering the mushy layer at $z = h$ has composition C_0 and temperature $T_0 = T_L(C_0)$, where $T_L(C) \equiv -mC$ is the liquidus temperature of the material being solidified and m is constant.

The liquid has dynamic viscosity μ, and the material has density ρ, specific heat c_p (both independent of phase) and latent heat of solidification L.

Write down a complete set of equations and boundary conditions describing a steady state of this system, ignoring the diffusion of solute and assuming that material properties are independent of phase. Determine the Darcy velocity in the mushy layer, assuming that its permeability is uniform.

Show that the equations admit a solution in which the temperature and solid fraction are independent of x.

Given that $\Pi E^2 h/\mu V \gg L/c_p(T_0 - T_E) \gg 1$, where Π is the permeability of the mushy layer, determine the approximate temperature field in the mushy layer. Hence or otherwise determine the composition of the solid product in terms of h as well as a transcendental equation for h.

3 A disk of mass M and radius a sits immersed in water, with axis vertical, on a horizontal plane, separated from it by a thin film of water. The temperature in the film is maintained at $T = T_0 + (T_m - T_0)r^2/a^2$, where r is radial distance from the axis, T_m is the bulk freezing temperature of water and $T_0 < T_m$ is a constant temperature. Ice forms in the film and premelts against the disk but not against the plane. Calculate the rate at which the disk separates from the plane. All symbols used in your answer should be defined.

What is the maximum weight that a disk of given radius can have and still be lifted?

Sketch a graph of the separation rate as a function of $T_m - T_0$. Give physical reasoning for the main features of the graph. What is the maximum rate of separation?

END OF PAPER