UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 8 June, 2006 9 to 11

PAPER 66

COMPUTER-AIDED GEOMETRIC DESIGN

*Attempt **FOUR** questions.*

*There are **SIX** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 P_1 and Q_1 are the endpoints of one line segment in 3D. P_2 and Q_2 are the endpoints of another. Describe an algorithm to determine the points R_1 and R_2, one on each segment, which are at a minimum distance from each other.

2 Describe an algorithm for tracing out as a sufficiently dense sequence of points, a single transversal intersection between a parametric surface (defined in the form $P = F(u, v)$, $u_{min} \leqslant u \leqslant u_{max}$, $v_{min} \leqslant v \leqslant v_{max}$) such as an untrimmed NURBS, and an implicit surface (defined in the form $f(P) = 0$) such as a sphere or torus.

3 The path of a virtual roller coaster at a website funfair is given as a parametric curve $P = F(u)$, $0 \leqslant u \leqslant 1$. The speed $v(P)$ at any point P of the path is a given function (determined from energy considerations).

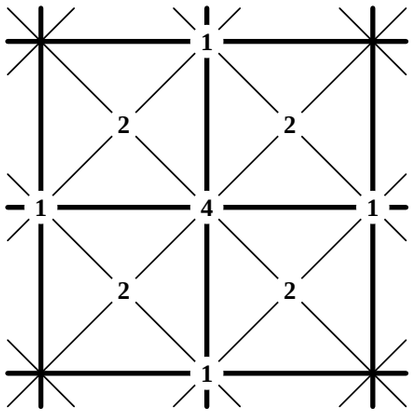
(a) How would you calculate a sequence of points at equal (short) time intervals ?

(b) Assuming that for safety reasons the accelerations are never enough to cancel out gravity exactly, show how to determine numerically at each such point the (orthonormal) axes of the camera coordinate system whose X axis is pointing along the direction of travel, with the locally felt 'up' direction (an appropriate combination of the acceleration and gravity) in its XZ plane.

4 (a) What enquiries should a subdivision curve definition be capable of supporting?

(b) Describe a recursive subdivision algorithm for drawing those parts of a curve which lie on the same side of a given plane $F(P) = 0$ as the eye-point E.

5 A subdivision surface is defined by a polyhedron and a refinement process whose mask (multiplied by 8) for a 4-valent vertex is



(a) Identify the stencils for the new vertex-vertices and the new face-vertices.

(b) Identify the mask of the binary scheme obtained by taking two steps at a time of this scheme.

(c) What is the shape and size of the region of surface influenced by moving a single control point?

(d) What are the box-spline factors of the z-transform of the binary mask?

(e) What degree of derivative continuity is achieved, between the polynomial pieces of what degree, in the regular parts of the limit surface?

(f) What criteria would you try to match if choosing the three mask coefficients for an extraordinary vertex of valency n? (Extraordinary faces disappear after one step of refinement.)

6 A long equal-interval quadratic B-spline curve is defined by the positions of the control points P_i of its control polygon.

(a) Determine the Bézier control points of the quadratic span influenced most by P_i.

(b) Determine the control points of the Bézier quartic obtained by degree-raising that span first to cubic and then to quartic.

(c) Determine the control points of the quartic B-spline (which has treble knots) representing the original curve. You should express these points in terms of the original control points P_i.

END OF PAPER