UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 12 June, 2006 9 to 11

PAPER 57

BRANES

Attempt ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Describe the relation between M-theory and: IIa string theory; IIb string theory on a circle; and the Heterotic string on T^3. In each case describe the M-theory compactification and how the moduli and branes in each theory are identified giving as much detail as possible.

2 Given that a supersymmetric brane action is invariant under the Fermionic gauge invariance called "kappa-symmetry", explain why the condition for a given brane configuration to preserve some fraction of supersymmetry is

$$\Gamma\epsilon = \epsilon,$$

and explain how this equation determines the fraction of supersymmetry preserved [You will need the form of the kappa symmetry transformation in terms of Γ but you are not expected to derive it]. What is the matrix Γ for the 11-dimensional supermembrane. Write down the κ symmetric action for the supermembrane. Given a Killing spinor then construct a calibration form. Describe what is meant by a calibration and show how a calibration leads to a minimal surface.

3 Write down the bosonic truncation of the action for a supermembrane coupled to the metric and 3-form potential A of 11-dimensional supergravity.

The M2-brane solution of 11-dimensional supergravity for metric and 4-form field strength F is

$$ds^2_{11} = H^{-\frac{2}{3}} ds^2 \left(E^{(1,2)}\right) + H^{\frac{1}{3}} ds^2 \left(E^8\right), \qquad F = \mathrm{vol}(E^{(1,2)}) \wedge dH^{-1},$$

where H is a harmonic function on E^8 with isolated singularities. Why does this imply the absence of a force between parallel membranes? Confirm this "no-force" condition by considering the action for a probe membrane in the M2 background.

Take the near horizon limit of the above membrane supergravity solution. Describe its geometry and establish the number of supersymmetries of the near horizon solution.

4

A D3-brane is described by a Dirac Born-Infeld action. What is the 1/2 BPS equation for the DBI theory? What is the BIon solution and how should it be interpreted? What is the region of validity for the solution? (You may assume a flat background.) Describe the supersymmetric solutions preserving less supersymmetry and how they can be interpreted in terms of D3-brane geometry.

END OF PAPER