UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 8 June, 2006 1.30 to 4.30

PAPER 54

SUPERSYMMETRY AND EXTRA DIMENSIONS

Attempt ***QUESTION 1*** *and any* ***THREE*** *of questions* ***2, 3, 4, 5***.

Question 1 carries 40% *weight; all other questions carry* 20% *weight.*

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Give an answer to each of the following questions in *no more* than two pages each:

(i) Write an essay about Kaluza-Klein theories and the Brane-world scenario. Include a discussion of the origin of the Kaluza-Klein tower, dimensional reduction and the relevant length scales.

(ii) Write a short essay about extended supersymmetries, including a discussion of BPS states. Explain clearly why extended supersymmetries are not expected to be relevant at low energies.

(iii) Write a short essay about the possible low-energy implications of $N = 1$ supersymmetry.

2 Derive the (anti)commutation relations of the $N = 1$ supersymmetry generators $Q_\alpha, \bar{Q}_{\dot{\beta}}$ with themselves and with the generators of the Poincaré group $M_{\mu\nu}$ and P_μ. Show how to construct the general massless multiplet for this algebra. Starting from the $N = 1$ supersymmetry algebra, prove that:

(i) In every supermultiplet, the number of bosons equals the number of fermions.

(ii) The energy is non-negative and supersymmetry is broken if the energy of the vacuum is strictly positive.

3 Write down the most general global $N = 1$ superspace action for interacting chiral and vector superfields of a simple gauge group, assuming no more than two derivatives appear in each term of the component Lagrangian. Identify clearly the Kähler potential K, the superpotential W, the gauge kinetic function f and the Fayet Iliopoulos constant ξ. By introducing 'spurion' fields X and Y in the F-term part of the Lagrangian, derive the general behaviour of each of the arbitrary functions K, W, f and parameter ξ above under quantum corrections. For simplicity restrict to a renormalisable theory. State clearly the use of holomorphy and symmetries in this proof.

4 Derive the dimension of the spinor representation in spacetimes of even dimensionality. Show that the 10-dimensional super Yang-Mills multiplet, consisting of one gauge field and its gaugino partner, reduces to $N = 4$ supersymmetric Yang-Mills in 4-dimensions. Starting from the pure Yang-Mills kinetic term in 10-dimensions, derive the scalar potential of the 4-dimensional theory. Look for the minima of this potential and show that there are flat directions.

5 Consider a chiral superfield Φ with components φ, ψ, F of charge q coupled to an Abelian vector field V with components A_μ, λ, D. Show that a non-vanishing vacuum expectation value of D, the auxiliary field of V, can break supersymmetry. Identify the corresponding goldstino field. Write down the renormalisable superspace action. Derive the part of the component Lagrangian that depends on the auxiliary field D. From this, derive the D-term part of the scalar potential. Find the condition on the charge q for supersymmetry to be broken. If supersymmetry is broken, find the mass splitting of the chiral multiplet. Under which condition is gauge symmetry broken? What happens if the Fayet-Iliopoulos term vanishes?

END OF PAPER