UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 7 June, 2005 1.30 to 3.30

PAPER 84

BOUYANCY EFFECTS IN FLUID

*Attempt **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Explain what is meant by the Boussinesq approximation and the strong version of the Boussinesq aproximation.

Fluid of density ρ and kinematic viscosity ν occupies an infinitely broad horizontal layer of thickness d. The bottom boundary of the layer is maintained at temperature $T_o + \Delta T$, while the top is maintained at temperature T_o.

Making the Boussinesq approximation, derive a governing differential equation for the vertical velocity of a linear motion in the layer. Also write down suitable boundary conditions if the surfaces are considered to be slippery and perfectly conducting.

Determine the maximum value of ΔT, say ΔT_{max}, for which this linear solution is valid. What happens for ΔT greater or less than ΔT_{max}?

2 A small source issues relatively light fluid of density ρ_o into a very large region of ambient fluid of density $\rho_a(z)$ with respect to a vertical z-axis. Making the Boussinesq approximation, define the specific mass, momentum and buoyancy fluxes, Q, M and F respectively.

Making the 'top-hat' approximation, in which values of all quantities are considered uniform across the resulting turbulent plume, derive differential equations which describe the variation of Q, M and F.

Consider now $\rho_a(z)$ to be a constant. Explain why the initial value of Q at the source, Q_o, can be neglected away from the source, indicating how far away from the source the neglect is valid.

Determine quantitative solutions for the two cases $Q_o = M_o = 0$ and $Q_o = F_o = 0$. Explain how to use these solutions if neither M_o or F_o are zero.

3 A thin, laminar high Rayleigh number, two-dimensional plume emanates from a horizontal line source at $x = z = 0$ in a porous medium that occupies the region $z > 0$. The density in the plume ρ is related to the temperature T by $\rho = \rho_0[1 - \alpha(T - T_0)]$, where ρ_0 is the reference density at the temperature T_0 at $x = \pm\infty$. On the assumption that the horizontal extent of the plume is very much less than its height and that the Boussinesq approximations are valid, show that the vertical velocity $w(x, z)$ is linearly proportional to $T(x, z)$.

Using a streamfunction representation, or otherwise, write down the governing differential equations that describe the behaviour of the plume.

Write down an expression for the vertically convected heat flux, F, in the plume. Using the governing equations, or otherwise, prove that F remains constant with height. Explain physically why this is so.

If the volume flux in the plume at $z = 0$ is zero, there is a similarity form of solution to the equations. Determine a suitable independent similarity variable, and the resulting similarity forms for the streamfunction and temperature fields. Use these to evaluate the variation of the volume flux with height in the plume.

Determine the resulting ordinary nonlinear differential equation and write down the relevant boundary conditions. Obtain the complete analytical solution of this system. Evaluate explicit relationships for the mass, momentum and buoyancy fluxes.

END OF PAPER