UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 9 June, 2005 9 to 12

PAPER 74

PHYSICAL COSMOLOGY

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (i) The Robertson-Walker metric

$$(ds)^2 = (c\,dt)^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

is the most general metric describing an expanding universe which obeys the Cosmological Principle. Use this metric to show that the proper area of a spherical surface, centred at the origin and passing through comoving coordinate r, is $4\pi[a(t)r]^2$.

(ii) Use the Robertson-Walker metric to show that the cosmological redshift, $z = (\nu_{\rm e}/\nu_0) - 1$, of a light source is related to the scale factor of the universe $a(t)$ by the relation

$$1 + z = \frac{a(t_0)}{a(t_e)}$$

where ν is the photon frequency, t is time, and the subscripts e and 0 denote respectively emission and reception.

Consider two galaxies, A and B. As viewed from Earth, galaxy A is at redshift $z_{\rm A} = 1$ and galaxy B is at $z_{\rm B} = 3$. What is the redshift of galaxy B as measured by a hypothetical observer on galaxy A?

(iii) Using the Robertson-Walker metric, show that, for a particle horizon to exist, the density of the dominant form of mass-energy at early times must scale as $\rho \propto a^{-\alpha}$ with $\alpha > 2$.

(iv) Hence show that in the radiation-dominated era there was a particle horizon, and that the proper distance to the horizon as a function of time was $s_{\rm hor}(t) = 2ct$. Give a brief physical explanation as to why $s_{\rm hor}(t) > ct$, where ct is the distance travelled by a photon in time t.

2 Write an essay entitled: "The Lyman alpha Forest: a Window on the Intergalactic Medium at High Redshift".

Your essay should include points (a), (b), and (c) below, plus at least one of (d), (e) and (f):

(a) A description, with an appropriate sketch, of what is meant by the term "Lyman alpha forest".

(b) Why do Lyman alpha forest absorption lines outnumber all other classes of QSO absorption lines?

(c) A summary of the main physical characteristics of Lyman alpha clouds. Explain, using simple physical arguments, why most Lyman alpha clouds are thought to arise in low density, highly ionised gas.

(d) A sketch of the distribution of neutral hydrogen column densities, $f(N_{\rm H\,I}, z)$ at $z \simeq 3$. Discuss how $f(N_{\rm H\,I})$ can be used to derive $\Omega_{\rm H\,I}$, the mass density of neutral gas expressed as a fraction of the critical density. Is $\Omega_{\rm H\,I}$ greater than, comparable to, or smaller than $\Omega_{\rm baryons}$? Discuss the implications of your answer.

(e) Does the number of Lyman alpha forest lines evolve with redshift? If so, in what sense? What redshift-dependent physical processes could cause changes in the Lyman alpha line density with redshift?

(f) How does the density of Lyman alpha forest lines change in the immediate proximity of a quasar? Explain how this effect has been used to constrain the intensity of the metagalactic ionising background.

3 (i) Show that the amplitude of a linear, super-horizon adiabatic fluctuation increases with time according to:

$$\delta = \delta_i \cdot \begin{cases} t/t_i & \text{in the radiation dominated era} \\ (t/t_i)^{2/3} & \text{in the matter dominated era} \end{cases}$$

(ii) The power spectrum of matter fluctuations, $P(k) \equiv \langle |\delta_k|^2 \rangle$, can be conveniently expressed in dimensionless form as:

$$\Delta^2(k) \equiv \frac{V}{(2\pi)^3} \, 4\pi k^3 P(k) \, .$$

Knowing that the correlation function is the Fourier transform of the power spectrum, show that a scale-free spectrum of primordial fluctuations of the form:

$$\langle |\delta_k|^2 \rangle \propto k^n$$

implies a power-law correlation function of the form:

$$\xi(r) = \left(\frac{r}{r_0} \right)^{-\gamma}$$

with $\gamma = n + 3$. What are the values of r_0 and γ measured for the present-day correlation function of galaxies?

(iii) Sketch the form of $P(k)$ over the range of values of k where its has been determined, indicating the observations which have contributed to its determination at the appropriate scales. Briefly describe the main features of $P(k)$ and comment on how they can be used to measure the cosmological parameters $\Omega_{m,0}$ and $\Omega_{\text{baryons},0}$.

4 (i) Show that the primordial abundance of helium by mass is

$$Y_{\rm p} = 2\left(1 + \frac{n_{\rm p}}{n_{\rm n}}\right)^{-1}$$

where $n_{\rm p}/n_{\rm n}$ is the ratio (by number) of protons to neutrons, if all the baryons are in H and He.

(ii) Imagine another universe, described by the same cosmological parameters as our own, but with the one difference that a force of unknown origin accelerated the universal expansion between times $t_1 = 1$s and $t_2 = 300$s. Would you expect the mass fraction of hydrogen in this alternate universe to be larger than, smaller than, or the same as, that in our universe? Justify your answer in two or three sentences.

(iii) "The observed abundances of the light elements are one of the pillars of the Big Bang theory: True or False?"

Your essay should include a discussion of the following points:

(a) A sketch of how the primordial abundances of the light elements depend on cosmological parameters. Why do some elements provide better constraints than others?

(b) How are the abundances of the light elements measured, and their primordial values deduced? What are (approximately) their primordial values?

(c) Are the values of the primordial abundances of the light elements as determined by observation consistent with each other? How do they compare with other independent measurements of the same cosmological parameters?

END OF PAPER