UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 14 June, 2005 9 to 11

PAPER 66

COMPUTER-AIDED GEOMETRIC DESIGN

*Attempt **FOUR** questions.*

*There are **SIX** questions in total.*

The questions carry equal weight.

To obtain full marks on a question all parts of that question must be answered.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (i) Prove that the first derivative with respect to t of a Bézier curve of degree n

$$P = \Sigma_{j=0}^{n} \frac{n!}{j!(n-j)!} P_j t^j (1-t)^{(n-j)}, t \in 0 \ldots 1$$

is an expression of the same form, one degree lower, whose coefficients are n times the first differences of the P_j.

(ii) What can you say about higher derivatives ?

(iii) What are the conditions on the defining polygons for continuity of first derivative between two Bézier curves defined over adjacent equal parametric intervals.

(iv) Using these ideas, derive a construction for the re-representation of a curve of degree 3 with given polygon as two pieces, covering $t \in 0 \ldots 1/2$ and $t \in 1/2 \ldots 1$.

2 (i) Two line segments, initially at A_0B_0 and C_0D_0, are moving in space without rotation and with constant velocities, the first with velocity V and the second with velocity W. Outline an algorithm for determining the earliest future time at which they make contact.

(ii) How is your chosen algorithm complicated if you want to find the first approach within distance d rather than actual contact ?

3 Given the mask $M_j, j = -w \ldots w$, and the arity a of a univariate subdivision scheme,

(i) Determine the support of the scheme (the width of the part of the limit curve which is influenced by one control point).

(ii) Identify two ways of calculating bounds on the level of derivative continuity of the curve with control points in general position, giving the gist of how each is justified.

(iii) Why does one of these give a lower bound and the other an upper bound on the level of continuity ?

4 Prove that the basis of a tensor product surface $P(u,v) = \Sigma_j \Sigma_k P_{jk} \phi_j(u) \psi_k(v)$ inherits each of the following properties from ϕ and ψ.

(i) positivity

(ii) summation to unity

(iii) level of derivative continuity

(iv) linear precision set

5 (i) What basic enquiries are required in order to support a wide range of geometric interrogations of parametric surfaces ?

(ii) Given the results of these enquiries, determine the tangent plane and Gaussian curvature at a given point of a parametric surface.

6 (i) What basic enquiries are required in order to support a wide range of geometric interrogations of subdivision curves ?

(ii) What basic enquiries are required in order to support interrogation of subdivision surfaces ?

(iii) Apply these enquiries in an algorithm to determine the shape of the shadow cast on a subdivision surface S by a subdivision curve C. You may assume that the curve does not intersect the surface, that it lies entirely between the surface and the light source, and that the surface is not so contorted that parts of it face away from the light source.

END OF PAPER