UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 13 June, 2005 9 to 12

PAPER 62

BLACK HOLES

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Write an essay describing in detail how one constructs the entirety of the spacetime associated with a single uncharged non-rotating black hole, given as your starting point the Schwarzschild metric. Describe qualitatively the consequences of allowing the hole to be electrically charged.

2 A static asymptotically flat black hole spacetime is given by the metric

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

As $r \to \infty, V(r) \to 1$ and $V(r) > 0$ for $r > r_0$. At $r = r_0$, $V(r)$ has a simple zero corresponding to a non-degenerate black hole horizon. By using Euclidean methods, derive the temperature T of the horizon.

The surface gravity κ of a black hole is defined by

$$k^a \nabla_a k_b = \kappa k_b$$

evaluated on the horizon, where k^a is the time translation Killing vector. Show that

$$|\kappa| = 2\pi T\,.$$

3 Sketch a proof of the black hole area theorem.

Describe the effect, over a long period of time, of Hawking radiation on the geometry of a uncharged non-rotating black hole and the physical consequences of this radiation.

How does the mass of an uncharged, non-rotating black hole evolve as a function of time?

Why does your result not contradict the area theorem?

4 A black hole is formed by the collapse of a spherically symmetric cloud of pressure-free matter. Initially the cloud is of uniform density and has a mass equal to that on the Sun, ($1M_{Sun} = 2 \times 10^{33}$ gm) a radius of 100 km, and is at rest.

What is the metric outside the collapsing matter?

What is the metric inside the collapsing matter?

An observer equipped with a stopwatch decides to follow the collapse, and moves with the outer boundary of the collapsing matter. At approximately what time (in seconds), according to this observer, does a horizon form? Approximately how much later (in seconds) according to this observer, does he reach the singularity?

A second observer watches this process from infinity. When does she see the horizon form?

You may use the following approximations:

$$\begin{bmatrix} 1 \text{ Planck mass} = 2 \times 10^{-5} \text{ gm} \\ 1 \text{ Planck time} = 5 \times 10^{-44} \text{ sec} \\ 1 \text{ Planck length} = 1.6 \times 10^{-33} \text{ cm} \end{bmatrix}$$

END OF PAPER