UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 9 June, 2005 1.30 to 3.30

PAPER 59

QUANTUM INFORMATION SCIENCE

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 The state $|\psi\rangle$ is an entangled pure state of qubits. Show that there exists a choice of axes for projective measurements on the two particles which violates the CHSH inequality.

2 State and prove the Schrödinger-Jaynes-Hughston-Jozsa-Wootters theorem classifying the ensembles of pure states corresponding to a given density matrix ρ.

3 The states $|\psi_1\rangle, \ldots, |\psi_N\rangle$ are linearly independent and are not all orthogonal. Is it possible to build a device which, if given as input an unknown choice $|\psi_i\rangle$ (where $1 \leqslant i \leqslant N$), always returns as output the value of i? Is it possible to build a device which, if given as input an unknown choice $|\psi_i\rangle$ (where $1 \leqslant i \leqslant N$), either (with some probability $p_i > 0$) returns as output the value of i, or (with probability $(1 - p_i)$) returns as output the declaration "state not identified". Justify your answers.

4 Alice creates two identical pure qubit states $|\phi\rangle$ and gives them to Bob, without telling him what the states are. Bob attempts to create a third qubit in the same state $|\phi\rangle$, and then returns all three qubits to Alice. Alice then applies projective measurements onto the state $|\phi\rangle$ to test that each of the three qubits is in the state $|\phi\rangle$. Show that there exists some probability $p > 0$ such that, whatever strategy Bob employs, the probability of at least one of the qubits failing Alice's tests is greater than p.

END OF PAPER