UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 10 June, 2005 1.30 to 4.30

PAPER 55

SUPERSYMMETRY AND EXTRA DIMENSIONS

*Attempt **QUESTION 1** and any **THREE** of questions **2, 3, 4, 5, 6**.*

Question 1 carries 40% weight; all other questions carry 20% weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Provide an answer to each of the following questions in *no more* than 25 lines each:

(a) Consider a massless representation corresponding to a theory with $N \geqslant 1$ supersymmetries. List the states, ordered by helicity λ, which can occur for a fixed value of the momentum. What is the total number of such states? Find the maximum number of supersymmetires allowed if $\lambda \leqslant 1$, or $\lambda \leqslant 2$. List all the states in these two representations and indicate how they can be derived from a higher-dimensional theory by dimensional reduction.

(b) Define what is meant by $R{-}parity$. Which problem of the minimal supersymmetric standard model (MSSM) is solved by introducing R-parity, and how is this solution achieved? Describe two physical implications of R-parity. Compare R-parity with R-symmetry.

(c) Write a short essay comparing the Kaluza-Klein scenario with both warped and unwarped Brane-World scenarios, which you should define. What is the largest value of the extra dimensions in each case? Explain in which sense these scenarios can address the hierarchy problem. How does that compare with the supersymmetry solution to this problem?

2 Write down the most general Kähler potential K and superpotential W for a renormalizable global $N = 1$ supersymmetric theory of a single chiral superfield Φ. Argue why a term in W linear in Φ can be eliminated by a field redefinition. Consider the quadratic and cubic terms in the superpotential and derive the auxiliary field part of the effective action. Solve the field equations for the auxiliary field and derive the corresponding scalar potential. Is supersymmetry broken in this model?

Taking the couplings g and m as spurion fields, define two $U(1)$ symmetries of the tree-level superpotential, with both $U(1)$'s acting on $\Phi,\ m,\ g$ and one of them acting also on the superspace coordinate θ (an R-symmetry). Use these symmetries to infer the most general form that the (loop corrected) effective superpotential can take.

3 Starting with the Maxwell action in five dimensions, explain how this can give rise to an infinite number of states in four dimensions after compactifying on a circle of radius r. Derive the expression for the masses of all the states and write down the low-energy effective action in four dimensions.

Explain the origin of internal symmetries from the Kaluza-Klein perspective. In particular, derive the gauge transformation of a vector field starting from the Kaluza-Klein ansatz in a five-dimensional gravity theory compactified on a circle of radius r.

4 Consider a chiral superfield Φ of charge q coupled to an abelian vector superfield V. Show that a nonvanishing vacuum expectation value of D, the auxiliary field of V, can break supersymmetry. Identify the corresponding goldstino field.

Write down the D-term part of the scalar potential and find the condition that the Fayet-Iliopoulos term and the charge q have to satisfy for supersymmetry to be broken.

Find the spectrum of this model after supersymmetry is broken. What is the mass splitting of the multiplet?

5 Consider $N = 1$ supergravity with two chiral superfields T, C. The Kähler potential is (in Planck units):

$$K \;=- \;3\;\log(T + T^*) + C^* C$$

The superpotential is:

$$W \;=\; C^3 + \;B$$

where B is an arbitrary complex number. Compute the scalar potential. Find the auxiliary fields for T and C and verify that supersymmetry is broken. What is the value of the vacuum energy at its minimum? Are there flat directions? What is the gravitino mass?

[*Hint: In supergravity the auxiliary fields are proportional to the Kähler covariant derivatives* $DW \;=\; \partial W/\partial\Phi + W\partial K/\partial\Phi$.]

6 Derive the $N = 1$ supersymmetry algebra:

$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha^\beta Q_\beta\,, \qquad [Q_\alpha, P^\mu] = \{Q_\alpha, Q_\beta\} = 0\,, \qquad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu\,.$$

where $M^{\mu\nu}, P^\mu$ ($\mu,\, \nu \,=\, 0, 1, 2, 3$) are the generators of the Poincaré group and $Q_\alpha, \bar{Q}_{\dot\beta}$ ($\alpha\,, \dot\beta = 1, 2$) are the fermionic supersymmetry generators.

Show that

$$\mathcal{P}_\mu = -i\partial_\mu$$
$$\mathcal{Q}_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu$$
$$\bar{\mathcal{Q}}_{\dot\alpha} = i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\gamma(\sigma^\mu)_{\gamma\dot\alpha}\partial_\mu$$

satisfy the $\{Q_\alpha, \bar{Q}_{\dot\beta}\}$ anticommutator in the $N = 1$ supersymmetry algebra above ($\theta^\alpha, \bar\theta^{\dot\alpha}$ are Grassmann coordinates).

END OF PAPER