UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 2 June, 2005 9 to 12

PAPER 27

SET THEORY

Questions in Part 1 are worth twice as many marks as questions in Part 2.

Full marks may be obtained by complete answers to (the equivalent of) ***FOUR*** *questions in Part 1.*

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

PART 1

1 Exhibit a recursive partition of $[\mathbb{N}]^3$ with no recursive monochromatic set. Prove the Erdös-Rado theorem on the existence of uncountable monochromatic sets for partitions of n-tuples. What can you say about infinite exponent partition relations?

2 Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

3 What is AD, the axiom of determinancy? Which games can you prove to be determinate? Establish that AD is inconsistent with AC.

4 Prove the independence of each of the following axioms from the remaining axioms of ZF: sumset, power set, replacement, extensionality, and infinity.

5 Write an essay on ultraproducts.

6 What is a measurable cardinal? An elementary embedding? Can there be an elementary embedding from the universe into itself?

7 What is a WQO? A BQO? State and prove Kruskal's theorem on wellquasiordering of trees.

8 State and prove a suitable generalisation of Cantor's Normal form theorem for ordinals.

9 Prove the consistency of NFU.

PART 2

10 An **incline** is a structure with two associative and commutative binary operations $+$ and $\cdot$ satisfying

(a) $(\forall xyz)(x \cdot (y + z) = x \cdot y + x \cdot z)$

(b) $(\forall x)(x + x = x)$

(c) $(\forall xy)(x + x \cdot y = x)$

We define a relation $\leq$ by $x \leq (x + y)$.

Let $\langle I, +, \cdot \rangle$ be a finitely generated incline. Show that $\langle I, \geq \rangle$ is a WQO.

11 Von Neumann's axiom states that every proper class is the same size as the universe. Prove that over Zermelo Set Theory it is equivalent to Replacement plus global choice.

12 State and prove Fodor's theorem.

13 Prove that if λ is singular strong limit cardinal then $2^{\lambda} = \lambda^{cf(\lambda)}$.

14 Prove that all values of the Hartogs' function are regular initial successor alephs.

END OF PAPER