UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 6 June, 2005 1.30 to 4.30

PAPER 17

TOPICS IN ALGEBRAIC GEOMETRY

*Attempt **ALL** questions.*

Question 1 carries 10 marks

and the others 30 marks each.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Define the formal completion $X_{/Y}$ of a scheme along a closed subscheme Y.

Suppose $X = \mathbb{A}^1_{k[q]} = \mathit{Spec}\ k[q,t]$ and $Z = \mathbb{G}_{m,k[q]} = X_1 - (t=0)$. Are the formal schemes $Z_{/(q=0)}$ and $X_{/(q=0)} - (t=0)$ isomorphic? Explain your answer.

2 Show how torus embeddings may be used to construct a morphism $X \to \mathit{Spec}\ \mathbb{Z}[q]$ such that the fibre Y over $(q=0)$ is an infinite chain of copies of $\mathbb{P}^1_{\mathbb{Z}}$ and the generic fibre is a copy of $\mathbb{G}_{m,\mathbb{Z}[q,q^{-1}]}$.

3 Take X, Y to be the schemes constructed in question 2. Explain how to construct a formal scheme that is the quotient of $X_{/Y}$ by some non-trivial action of an infinite cyclic group.

4 Suppose that M is a free $\mathbb{Z}$-module of finite rank. Show that the functor $F : \mathit{Rings} \to \mathit{Sets}$ such that $F(R)$ is the set of locally free rank r quotient modules of $M \otimes_{\mathbb{Z}} R$ is representable.

END OF PAPER