UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 27 May, 2004 1.30 to 3.30

PAPER 70

COMPUTER-AIDED GEOMETRIC DESIGN

Attempt **FOUR** *questions.*

There are **six** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (i) Given four points, A, B, C and D, in general position in E3, describe how to compute the position of the circumcentre of the tetrahedron $ABCD$. Although a single closed form expression is not required, the individual 'assignment statements' of your 'program' should be explicit computations.

(ii) Similarly, describe how to compute the incentre of the tetrahedron.

2 (i) Define the univariate Bernstein-Bezier basis functions of degree n. What properties make them attractive in a CAGD context ?

(ii) Why are the B-splines even more attractive ?

(iii) From the definition of the B-spline in terms of minimum support and maximum continuity, determine the equal-interval cubic B-spline basis function as a sequence of cubic Bezier pieces. Show your working.

3 The Chaikin curve subdivision scheme starts from a polygon and constructs a new polygon by putting new vertices at the 1/4 and 3/4 points of each old edge, and joining them in the obvious sequence to form a new polygon. Iterating this procedure gives a sequence of polygons which converges to a curve. Consider the case where vertices of the initial polygon are equally spaced in x, but have individual y coordinates, y_i.

(i) How much of the curve is influenced by a change in one original polygon vertex ? Show your working.

(ii) For what values of n is it true that if all the vertices lie on a polynomial of degree n in x at one stage, all the vertices will lie on a polynomial of degree n in x at the next stage ? Show your working.

(iii) How many derivatives are continuous at the centre of each original edge ? Show your working.

(iv) Thence determine the explicit form of the basis function. Show your working.

4 (i) What is a tensor product parametric surface definition ? What does this imply for the control polyhedron ?

(ii) Show that the tensor product B-spline surface basis functions have continuity of the same order of derivative as the univariate functions from which they are constructed. What other properties are inherited from the univariate functions ?

(iii) Give an example of a parametric surface basis which is not a tensor product.

5 (i) What information must a parametric surface be able to provide to program code for computing properties of that surface ?

(ii) Describe an algorithm which, given the position in E3 of P, a point light source, C, a parametric curve, and S, a parametric surface, will compute a sequence of points on S lying in the shadow cast by C. The sequence should be dense enough so that joining the points by straight lines gives an approximation within a reasonable tolerance for graphics purposes. You may assume that S faces P, so that in the absence of C all of one side of it would be lit, and that there exist two parallel planes, dividing space into three parts, one containing P, one containing S and one in between containing C.

(iii) What configurations would you need to allow for if you could not make these assumptions ?

6 (i) What information must a recursive subdivision surface be able to provide to program code for computing properties of that surface ?

(ii) Describe an algorithm which, given two subdivision surface instances which are known to be closed and $C1$, determines the minimum distance between the bodies enclosed by these surfaces.

You may assume that the polyhedra describing the surfaces are disjoint, so that the distance is positive.

(iii) How would your algorithm work without the assumption of disjointness ? How would you need to change it to allow for overlap, when the distance should come out negative ?